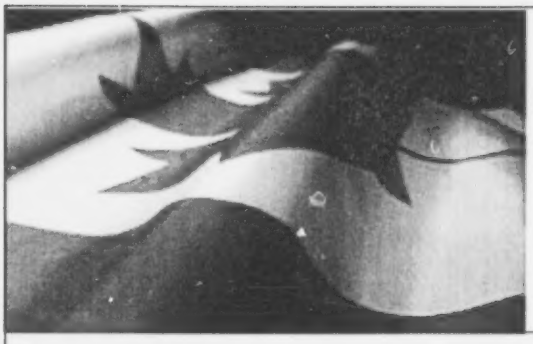

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The Value of Waiting and the Theory of Investment Under Uncertainty (With an Application to Kyoto)

by

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* The views expressed in this paper are those of the author and do not necessarily reflect those of the Department of Finance. I would like to thank Benoît Robidoux, Aled ab Iorwerth, and Chris Matier for helpful comments and discussions. All remaining errors are my own.

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Abstract

This paper applies real option theory to demonstrate that increased uncertainty in the economic environment due to the Kyoto protocol generates additional inertia in investment decisions which are not taken into account in "traditional" (*e.g.*, Computable General Equilibrium) models. Thus, if Kyoto were to increase only the volatility of cash flows (revenues), the modeller who neglects the option value of waiting, inherent in investment decisions under uncertainty, would tend to *underestimate* the negative impact of Kyoto on investment.

On the other hand, if Kyoto implies a fall in the level of expected revenues, a modeller who neglects the option value of waiting would tend to *overestimate* the negative impact of Kyoto on investment. If Kyoto is modelled as a mix of lower level of expected revenues and higher volatility of revenues, it could be the case that the two types of errors due to the omission of the option value of waiting cancel out, however, this would generally not be the case.

Résumé

Ce papier utilise la théorie des options réelles pour démontrer qu'un environnement économique plus incertain dû au Protocole de Kyoto pourrait générer des inerties supplémentaires dans les décisions d'investissement qui ne sont pas prises en compte dans les modèles traditionnels (*e.g.*, modèles d'équilibre général calculable). Le modélisateur qui néglige la valeur d'option d'attente inhérente aux décisions d'investissement sous incertitude, sous-estimerait l'impact négatif de Kyoto sur les investissements, si le seul impact de Kyoto était d'accroître la volatilité des revenus.

D'autre part, si Kyoto implique une baisse des revenus attendus, un modélisateur qui néglige la valeur d'option d'attente surestimerait l'impact négatif de Kyoto sur l'investissement. Si Kyoto est modélisé comme une combinaison de niveau de revenus plus bas et de volatilité de revenus plus élevée, il se peut que les deux types d'erreurs s'annulent, mais ce ne sera généralement pas le cas.

1. Introduction

The majority of scientists agree that there is evidence of a discernable human influence on the global climate from the accumulation of greenhouse gases (GHG) in the atmosphere. This finding led to the Kyoto Protocol, under which developed countries are required to reduce their GHG emissions to an average of 5% below their 1990 emission levels by 2008 to 2012. Canada's target under the Protocol is to reduce its GHG emissions to 6% below its 1990 levels.

In 2000, Canada's GHG emissions were about 20% above 1990 emission levels and are forecast to increase further by 2008-2012, making emission reductions a significant challenge. The Canadian Plan on Climate Change proposes a set of initiatives to take up that challenge.[1] With Canada's ratification of the Kyoto Protocol in December 2002, the Government is moving toward implementing its Climate Change Plan. Addressing the environmental challenge of climate change has direct economic implications since most of the GHG emissions result from the production and consumption of fossil fuels, which are omnipresent in our society.

The objective of this paper is to highlight some key points that have been and will be of importance to large final greenhouse gas emitters when discussing the impact of Kyoto on their investment behaviour. In particular, the paper applies the theory of the value of waiting to demonstrate that increased uncertainty in the economic environment due to the Kyoto protocol generates additional inertia in investment decisions which are not taken into account in "traditional" (*e.g.*, Computable General Equilibrium) models. Thus, if Kyoto were to increase only the volatility of cash flows (revenues), the modeller who neglects the option value of waiting would tend to *underestimate* the negative impact of Kyoto on investment opportunities.[2]

On the other hand, if Kyoto implies a fall in expected revenues, a modeller who neglects the option value of waiting would tend to *overestimate* the negative impact of Kyoto on investment. If Kyoto is modelled as a mix of lower expected revenues and higher volatility, it could be the case that the two types of errors due to the omission of the option value of waiting cancel out, however this would generally not be the case.

The plan of the paper is as follows. Section 2 explains the theory of the value of waiting in general terms. Section 3 introduces a heuristic approach to some quantitative aspects of this theory. Section 4 uses the theory developed in Section 3 to demonstrate how Kyoto could affect investment decisions beyond and above the projections of CGE models. This section also provides some direction on how to introduce the value of waiting into a CGE model. Finally, concluding comments are provided in Section 5.

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[1] The plan is available at: http://www.climatechange.gc.ca/plan_for_canada/plan/pdf/full_version.pdf

[2] However, the policy response of the government (*e.g.*, price cap on permit prices and free allocation of a share of total permits) is shown to lower the additional uncertainty due to Kyoto and thus has the potential to mitigate the concerns of the GHG emitters.

2. Investment under Uncertainty

Once upon a time in New York City, there lived an Assistant Professor of Finance. She and her spouse-equivalent had separate apartments. Their relationship progressed to a point where her spouse suggested that they should keep one of the apartments and give up the other. She then explained to him the importance of keeping options open; it was unlikely that they would split up, but given a positive probability…

This banal example, adapted from Dixit (1992), may be viewed as a non-economic application of the value of waiting. In economic terms, the literature on investment under uncertainty has established that having the option of waiting before investing is valuable under specific conditions.

The traditional (or, original) economic theory of investment rests on the foundations of Marshall's analysis. If price exceeds long-run average cost, this induces existing firms to expand, and new ones to enter. If the price falls below average cost, then firms suspend operations or exit from the market altogether.

However, reality is often different. Firms invest in projects they expect to yield a return in excess of a required or "hurdle" rate. Firms do not invest until the price rises substantially above long-run average cost. Some developments in the theory of investment under uncertainty have offered an interesting explanation of these phenomena, suggesting that a great deal of inertia is optimal when decisions are being made in an uncertain environment.

Timing of Investment and the Value of Waiting

Three features are common to most investment decisions and they combine to yield inertia in investment decisions. First, an investment entails some sunk cost, that is, an expenditure that cannot be recouped if the action is reversed at a later date. Second, the economic environment has ongoing uncertainty, and information arrives gradually. Finally, an investment opportunity does not generally disappear if not taken immediately; the decision is not only whether or not to invest but also when to invest.

The qualitative implication is easily stated. When these three conditions are present, waiting has positive value. In an evolving environment, time brings more information about future prospects of a project. As long as the opportunity to invest remains available, a later decision can be a better one. And because there are sunk costs, it does not always pay to take a less perfect action now and then change it later.

The main point is that waiting for a certain amount of time enables an investor to avoid the downside risk in revenues over that interval, while realizing the upside potential. This selective reduction in risk over time generates a positive value of waiting. The literature has established this positive value under the assumption of risk-neutrality. The

purpose of this assumption is to show that the value of waiting has nothing to do with risk-aversion.

Of course, the value of waiting must be set against the sacrifice of potential revenues that would materialise from investing. If current conditions become sufficiently favourable to the investment, one should eventually take the action that is optimal, and not wait any longer. The trigger level that makes it optimal to proceed exceeds the "Marshallian" normal return. This view of investment says that firms that refuse to invest even when the currently available rates of return are far in excess of the cost of capital may be waiting optimally in order to be more certain that these conditions are not transitory.

Timing of Disinvestment

The possibility of waiting influences the decision to abandon projects, just as it affects the decision to invest, which suggests that inertia underlies disinvestment decisions. The typical textbook Marshallian theory suggests that disinvestment should take place when operating losses are being made. In reality however, the investor is willing to tolerate some operating loss to keep alive the option of future profitable operation. Indeed, if a firm ceases operation, it cannot restart at will without incurring some additional costs (i.e., it is as if the machinery rusts when unused). If a firm ever wants to restart in the future, the whole sunk cost must be incurred again.

In conclusion, a firm that refuses to exit even when the currently available rates of return are less than the cost of capital may also be optimally waiting before exiting because the value of the option of future profitable operation exceeds the current loss.

3. A Quantitative Evaluation of Inertia in Investment Decisions

The argument above is purely qualitative; it says that waiting has a positive value, but not whether this value is typically large. Dixit, however, shows that for "reasonable" parameter values, the potential revenues of a project would have to rise to nearly double the level that ensures a positive net worth before waiting ceases to be optimal. The objective of this section is to introduce a heuristic approach to help appreciate this quantitative statement. The mathematics of the value of waiting is relegated to an Appendix.

The Example of an Investment Project

The methodology used here is to describe the most simple framework of an investment decision (a risk neutral firm who has a monopoly right to invest in a given project and thus ignoring the interaction with other firms) and show how ongoing uncertainty, sunk cost, and the possibility of investing now or later eventually give rise to a value of waiting. The framework may, however, be extended to take into account, among other

things, risk aversion, competitive industry, incremental investment, and (convex) adjustment costs. Dixit and Pindyck (1994) is the key reference in the field.[3]

Consider an investment project, say, an oil well, which can be launched by incurring a sunk cost, K, and once launched, last forever. Let R denote its cash flow of net operating revenues per unit of time (say, per year) as shown in the second row of Table 1. The present value of this project is easily computed as R/r (perpetuity) where r is the discount rate, or opportunity cost of capital.[4] For the investment opportunity to be implemented, it must be the case that:

$$NPV = R/r - K > 0 \tag{1}$$

where $R/r - K$ is the net present value of the investment (NPV).

The net present value indicates the additional value that this project brings to the firm, assuming that the funds, if not used for this project, would have been invested elsewhere in a project belonging to the same class of risk. The discount rate, or opportunity cost of capital, *implicitly* takes into account this alternative. As the project considered in Table 1 is assumed risk-free, the appropriate discount rate must be the risk-free interest rate. This means that the alternative to the oil well is implicitly viewed as investing an amount of money K at the "normal" risk-free rate of return r, generating an annual (non-reinvested) cash flow of interest revenues, rK, as shown in the third row of Table 1.

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[3] Dixit A., and R. Pindyck (1994), "*Investment Under Uncertainty*", Princeton University Press, Princeton (1994).

[4] The present value (to Year 0) of the annual flow of operating revenues in Table 1 is given by:

$$PV = \frac{R}{1+r} + \frac{R}{(1+r)^2} + \frac{R}{(1+r)^3} + ... = \frac{R}{1+r}\underbrace{\left(1 + \frac{1}{1+r} + \left(\frac{1}{1+r}\right)^2 + ...\right)}_{=1/\left(1-\frac{1}{1+r}\right)\,(\text{if } 1/(1+r)<1 \Rightarrow \text{if } r>0)} = \frac{R}{1+r}\frac{(1+r)}{(1+r-1)} = \frac{R}{r}$$

Table 1. Annual flow of operating revenues of an investment opportunity

Time Horizon	Year 1	Year 2	Year 3	...		Present value	Net present value
Annual cash flows of net operating revenues	R	R	R	...		R/r	R/r-K
Alternative cash flows	rK	rK	rK	...		K	0
Marshallian's critical value, R=1; r=0.05; K=20	1	1	1	...		1/0.05	0

Comparing the second and third rows, it is obvious that if the annual cash flow of the oil well project, R, is just equal to the alternative rK, the investor is indifferent between investing in the project or not (*i.e.*, maintaining the current alternative). If $R > rK$, the project has an annual rate of return R/K that exceeds the opportunity cost of capital, r (or "normal" rate of return on a risk-free investment) and the firm should invest in the oil well project. It should be clear that this leads to the same criterion as the one proposed by the net present value formula given in equation (1).

There exists a level of net revenue R such that the net present value is equal to zero (and this level is the same as the one that equalises the cash flows of the project with those of the alternative). As an example, assuming a discount rate of r=0.05 and a sunk cost of K = 20, an annual flow of net revenue of $R = rK = 1$ would make the firm indifferent between investing or not, and we refer to this level of net revenue as the Marshallian critical value.

If the stream of net revenue R exceeds 1, an investment is profitable according to the traditional theory. Let us assume that it is the case. Suppose therefore that R slightly exceeds 1. Does this mean that it is in the interest of the firm to invest immediately? The answer is negative in an environment of uncertainty.

Introducing Uncertainty and Risk Neutrality[5]

To see why this is the case, suppose that future revenues are only imperfectly predictable from the current observation ($R = 1$). The probability distribution of future net revenues is determined by the present, but the actual path remains uncertain. The probabilistic law of evolution of R can take many forms. We suppose that each period, R can either

[5] Appendix A1 provides a very simple numerical example where uncertainty lasts only two periods.

increase or decrease by a fixed percentage. Figure 1 illustrates a binomial tree that is a discrete-time and discrete-state approximation of the so-called geometric Brownian motion (used in Appendix A2).[6] Many economic time series, including the prices of natural resources or stocks, can be described as following such a process. [As an example Figure 2 shows the daily price of oil in year 2001 and one realisation artificially generated from a process that follows a geometric Brownian motion.]

The binomial tree in Figure 1 bends due to the "geometric" nature of the process.[7] The top and bottom branches of the tree in Year 5 can only be reached by one path each, either with all 'up' or with all 'down' moves, whereas there will be several paths possible for each of the intermediate values in Year 5; (there exist six different paths which lead to the node $R = 1$ in year 5, whereas only four paths lead to $R = 1.49$ or 0.67). Therefore, the intermediate values are more likely to be reached than the end values. The binomial tree contains within it an approximation to a log-normal probability density function.[8]

We assume in Figure 1 that the current (first year) net revenue of the project is equal to R=1 (the Marshallian critical value derived above). The binomial tree is built to ensure that the expected value of the net revenue for each future year, conditional on the current

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[6] The binomial tree requires a starting point, R, from which the level of revenue can move up to Ru with probability p and down to Rd with probability 1-p in a small interval of time Δt. The parameters u, d, and p must be chosen to give the binomial model the same drift and standard deviation than if it was a geometric Brownian motion [that is: $E(R_T) = \mathrm{R}e^{\mu(T-t)}$ and $Var(R_T) = R^2 e^{2\mu(T-t)}(e^{\sigma^2(T-t)} - 1)$]. Having only two restrictions and three parameters to choose, we have one degree of freedom. This degree could be used to choose $p = ½$, so that the probability of an up move is set equal to the probability of a down move. Alternatively, we can impose that after an up and a down movement, the revenue will return to its initial level so that ud=1. Imposing this additional restriction it can be shown that for a small interval of time T-t=Δt, we have that $u = e^{\sigma\sqrt{\Delta t}}$; $p = \dfrac{e^{\mu\Delta t} - d}{u - d}$; where σ and μ are the volatility and drift of the geometric Brownian motion followed by R [see equation (1) in Appendix A2]. Both parameters σ and μ will be shortly explained in further details in the text.

[7] For example, the upper path going from Node 1 in Year 1 to Node 2.23 in Year 5 forms a geometric series given by: R, Ru, Ru^2, Ru^3, Ru^4, where R = 1 and u = 1.22. For the lower path, the series is R, Rd, Rd^2, Rd^3, Rd^4, where R = 1 and d = 1/u = 1/1.22 = 0.82.

[8] A variable has a lognormal distribution if the natural logarithm of the variable is normally distributed. In the case of the geometric Brownian motion, it can be proven that: $\ln R_T \sim \mathrm{N}\left(\ln R + (\mu - \sigma^2/2)(T-t), \sigma\sqrt{T-t}\right)$ where R_T is the revenue at time T, R is the revenue at the current time t, and N denotes a normal distribution. A variable that has a lognormal distribution can take any value between zero and infinity. Given the normal distribution followed by $\ln R_T$, it can be shown [see Ramanathan (1993)] that the expected value of R_T is given by: $E(R_T) = \mathrm{R}e^{\mu(T-t)}$ and the variance of R_T is given by: $Var(R_T) = R^2 e^{2\mu(T-t)}(e^{\sigma^2(T-t)} - 1)$. Hence, when there is no drift, that is when $\mu = 0$, $E(R_T) = R$. If the binomial tree were constructed with log values of R, it would be symmetric around $\ln R = 0$ (when there is no drift) instead of bending as it is with *level* values of R.

revenue, is also equal to 1 (see footnote 8). We can verify this intuitively by recalling that Node $R = 1$ in Year 5 is more likely than the alternatives. This implies that most of the (log-normal) mass distribution of R in year 5 is around 1. We can compute that the probability of reaching Node R=1 at Year 5 is about 37%, while reaching Node 2.23 is a mere 4% as illustrated in Figure 3.

The net present value of the project in Figure 1 is a random variable. We assume that the investor is risk-neutral, so that the variance of NPV is irrelevant to him. Risk-neutrality is assumed here to illustrate that the concept of value of waiting has nothing to do with risk-aversion. Given that the expected value of future flows of revenues conditional on the information in Year 1 is equal to 1, and assuming, as before, a discount rate of $r = 0.05$ and a sunk cost of $K = 20$, the expected net present value of the project is equal to 0.[9] Under risk-neutrality, the project in Figure 1 is thus equivalent to the project initially considered in Table 1.

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[9] The discount rate used in the net present value formula is the normal rate of return of an alternative investment opportunity within the same class of risk. Although the project considered in Figure 1 is risky (uncertain future net revenues) assuming risk neutrality justifies the use of the risk-free interest rate in the present value formula.

Figure 1. A Binomial Model: mu = 0, sigma = 0.2

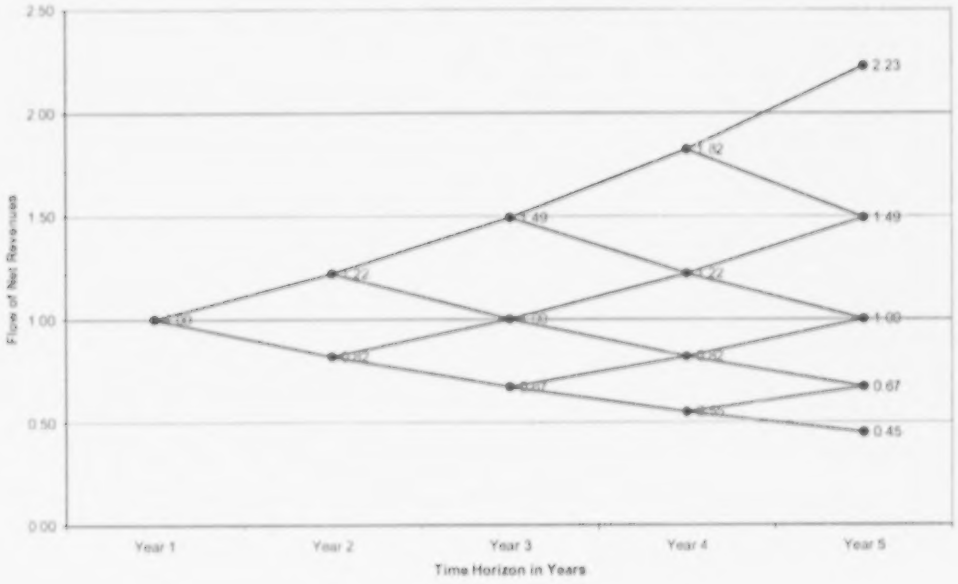
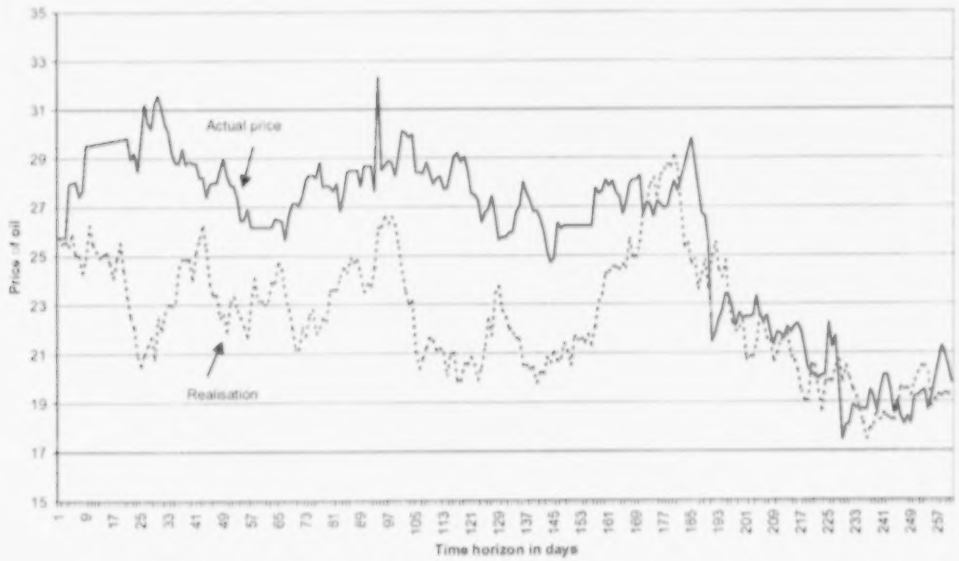
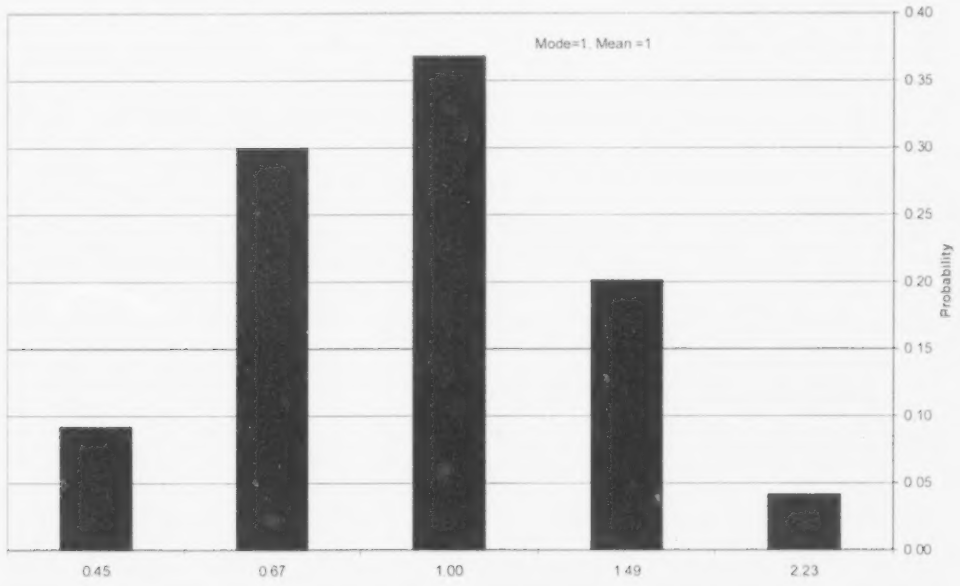


Figure 2. Daily Prices of Oil in 2001 versus a Realisation from a Process that Follows a Geometric Brownian Motion



Even in a context of risk-neutrality, uncertainty has important implications if waiting is an option. To see this, consider the following strategy. Being at Node R=1 in Year 1, the investor waits for a period of two years and observes the value of the net revenue flow in year 3. If negative shocks lead to Node R = 0.67 in Year 3, this means that the *conditional* expectation of the annual flow of net revenue has now fallen to 0.67. The net present value of the project conditional on this expectation is now negative ($NPV = \frac{0.67}{0.05} - 20 = -6.6$), which implies that the project will not be implemented.

Figure 3. Log-normal probability distribution of operating revenues: mu=0, sigma=0.2



Alternatively, the rate of return of the project has fallen to R/K = 0.67/20 = 3.3%, smaller than the opportunity cost of capital, 5%. Thus by waiting, the investor has avoided the mistake of investing in a project that initially had a rate of return just equal to the cost of capital but now offers a rate of return well below the normal rate of return. On the other hand, if positive shocks lead to an upward revaluation of the project, e.g., at node R = 1.49 in Year 3, the investor becomes more certain that the initial project is indeed a good opportunity ($NPV = \frac{1.49}{0.05} - 20 = 9.8$, and rate of return = R/K = 7.45%).

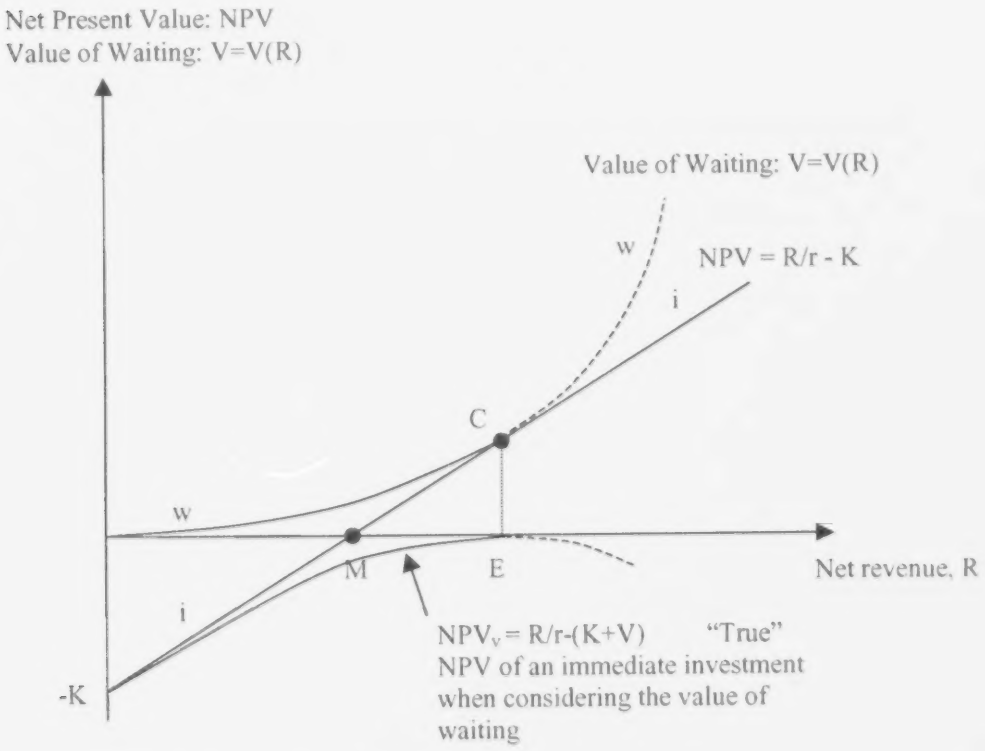
The point is that waiting for a certain amount of time enables an investor to avoid the downside risk in revenues over that interval, while realizing the upside potential. Hence, waiting is valuable. This should not be confused with the statement that uncertainty is

"bad" in itself, which is not what this theory implies (recall that we assumed that the investor is risk-neutral); it is the selective reduction in risk over time that generates a positive value of waiting.

On the other hand, the cost of waiting is the sacrifice of the profit flow over the period of waiting. Therefore, if the current net revenue flow reaches a sufficiently high level, it would not pay to wait any longer. There is a critical level, say, E, such that investment is optimal when current revenue exceeds it. But this level E, is higher than the Marshallian level, M, which creates inertia in investment decisions.

We present the cost-benefit analysis graphically in Figure 4. The straight line, NPV, represents the net present value of the investment as defined in equation (1) for each level of current net revenue given on the horizontal axis. When R exceeds M (= Kr) the investment is profitable.[10] Let us assume that this is the case. Suppose therefore that R slightly exceeds M. Does this mean that it is in the interest of the firm to invest immediately? The answer is negative.

Figure 4: The option value of waiting



[10] By setting $R = Kr$ ($=M$) in equation 1 we obtain that $NPV = 0$.

Point M in Figure 4 is the critical value of the "Marshallian" net present value rule. However, being at M is not a sufficient condition for the firm to invest. The Marshallian rule of net present value is based on the assumption that the choice facing the firm is between investing *now* (to get $R/r-K$) or *never* (which gets 0). What happens if it is possible to wait for a while and then reassess the decision?

If the firm has the option to wait before making its final decision, the cost-benefit calculus changes. Now, at the Marshallian trigger, waiting is better than either investing right away or never investing.

For example, consider one *possible* (but not necessarily optimal) waiting strategy. Assume that we are at the Marshallian trigger ($R = M$), waiting for a fixed interval of time, and observe the value of R, say R_1, at its end. If and only if revenue $R_1 > M$, investment would take place (in all other cases it is assumed that we never invest). If the revenue at the end of the waiting period exceeds the Marshallian trigger ($R_1 > M$), then the net value of the investment must be positive at that time, and remains positive when discounted back to the starting time. If revenue is less than the Marshallian trigger ($R_1 < M$), the net worth is zero because no investment is undertaken. The expected value of a positive number and zero is positive and therefore this waiting strategy has a positive value when discounted back to the present. Therefore the proposed waiting strategy does better than either investing right away or never investing, each of which yields zero when current revenue is exactly equal to the Marshallian trigger.

We show the value of waiting by the curved line *w-w*. When $R = M$, the value of waiting is positive and is represented graphically by the (positive) height of the curve *w-w* at $R = M$. By continuity, waiting remains better than investing for values of R slightly in excess of M.[11] Thus even if today R slightly exceeds M, it pays for the firm to wait. If $R < E$, the rule tells us that the firm should wait before investing. Only when R exceeds E is it optimal for the firm to invest immediately. Put differently, the expected revenue must exceed the minimum required revenue by a margin. The schedule NPV_v in Figure 4 is explained in the next subsection.

Including the value of waiting in the traditional net present value formula

Section 4 illustrates in more detail how the theory of the value of waiting could be used to provide a rationale for statements claiming that the Kyoto protocol and its recent ratification by the Canadian Parliament would substantially impact the level and timing

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[11] The intuition behind the general form of w-w is as follows. The value of waiting should approach zero if the current R is very low, because then the event of R climbing to E is unlikely except in the far future, and the discounted present value of that is quite small. Successively higher values for R should raise the value of waiting rapidly. For R close to E but just below it, the probability of reaching E in the near future approaches one, and the option value of waiting approaches the net present value of investing. Also, an optimal E implies that w-w should be tangent to i-i (the smooth-pasting condition). See Appendix A2 for a derivation of this curve.

of investment in Canada. Before demonstrating this, we show how the net present value formula should be adapted to take the value of waiting into account in an investment decision.

The implication of the theory of investment under uncertainty is that immediate action for the firm has an opportunity cost, the loss of the option to wait. Let us call this cost, $V \geq 0$. We must add this to the actual sunk cost, K, to get the full cost of immediate investment. Then, such action is justified as soon as the present value of future revenues of the investment equals the full cost of immediate investment. This is observed when:

$$\frac{R}{r} = V + K .$$

or:

$$NPV_v = \frac{R}{r} - (K + V) = 0 .$$

This suggests that the net present value of the traditional theory as given in equation (1) and rewritten here as:

$$NPV = \frac{R}{r} - K ,$$

overestimates the "true" net present value of an investment which accounts for the loss of the option to wait, and is given by:

(2) $$NPV_v = \frac{R}{r} - (K + V) .$$ [12]

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[12] The overestimation of the net present value in the traditional theory is thus $NPV\text{-}NPV_v = V$. The value of waiting is assumed to be a function of R, such that $V=V(R)$. Equation (20) of the Appendix gives the analytical expression of V as a function of R. It can be shown that $\lim_{R \to 0} V(R) = 0$ and that $\lim_{R \to E} V(R) = \frac{E}{r} - K$ (when the trend $\mu=0$). Hence, for very small values of R, the overestimation of the net present value in the traditional theory tends to zero, whereas as R increases, the overestimation grows and eventually reaches a maximum at the optimal trigger, E, and given by: $\frac{E}{r} - K$. Alternatively, at the optimal trigger E, $NPV = \frac{E}{r} - K$ whereas the net present value adjusted for the value of waiting

Figure 4 above plots both the traditional and adjusted net present value curves. The overestimation of the net present value implies that investment is typically overestimated in the traditional theory because it does not take into account the option value of waiting that occurs in an uncertain environment (even when agents are risk neutral). We see that at the optimal trigger E, the adjusted net present value is equal to zero.

Does Inertia Really Matter?

We have explained that there is a value of waiting and thus, that there is inertia in investment decisions. This inertia is not appropriately measured in traditional theories of investment. But does it really matters empirically? In other words, is the distance between the Marshallian and optimal critical values large? Of course the answer depends on the "parameters" of the model. Dixit shows that for "reasonable" parameters values, the net revenues of a project would have to rise to nearly double the level that ensures a positive net worth before waiting ceases to be optimal.

In order to understand why this is the case, we need to describe some of the parameters underlying Figures 1 (above) or 5 (below). A key parameter of the model is the volatility, a measure of dispersion. For an investment project like an oil well, revenues will fluctuate with the price of oil. Box 1 shows how volatility is actually estimated from historical data on oil price. It is shown that volatility in oil markets was about 47% p.a. during 2001, up from 31% p.a. in 1997. Figure 1 is calibrated with a volatility of 20% p.a. As a comparison, Figure 5 is calibrated with a volatility of 30%. Thus, relative to Figure 1, we observe in Figure 5 that extremes nodes are spread farther apart for each successive year.

A second parameter underlying Figures 1 and 5 is the revenue trend of a project. In those figures the trend is set equal to zero, which implies that the conditional expectation of the revenues in future years remain equal to its current value. We could however impose a positive trend to the project to ensure that the expectation of future revenues increase through time. In terms of Figures 1 and 5, this would translate into a higher probability of an increase than a decrease at each node. With volatility and trend respectively set at 0.20 and 0.03, Figure 1 would remain unchanged but its corresponding probability distribution in year 5 would now be given by Figure 6. The mean of this distribution in year 5 is actually 1.13. Without uncertainty this project would have the annual flow given in Table 2 and we observe that the investor would now be ready to incur a sunk cost of K= 50 in order to obtain this flow of revenues.

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is: $NPV_v = 0$. Given equation (19) in the appendix, $E = rK\left(\frac{\lambda_2}{\lambda_2 - 1}\right)$, and thus the overestimation of the net present value at E is also given by: $V(E) = K\left(\frac{1}{\lambda_2 - 1}\right)$

Figure 5. A Binomial Model: mu = 0; sigma = 0.3

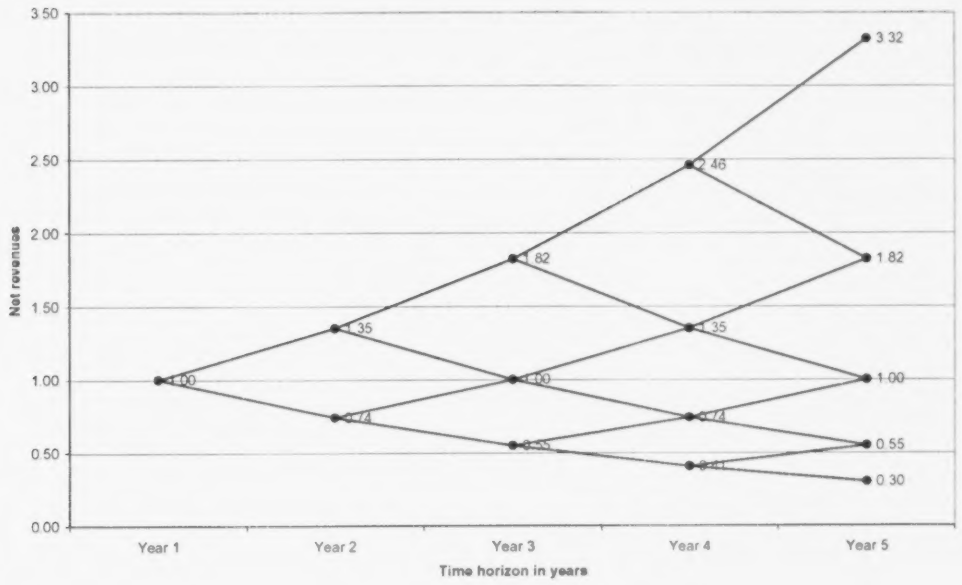
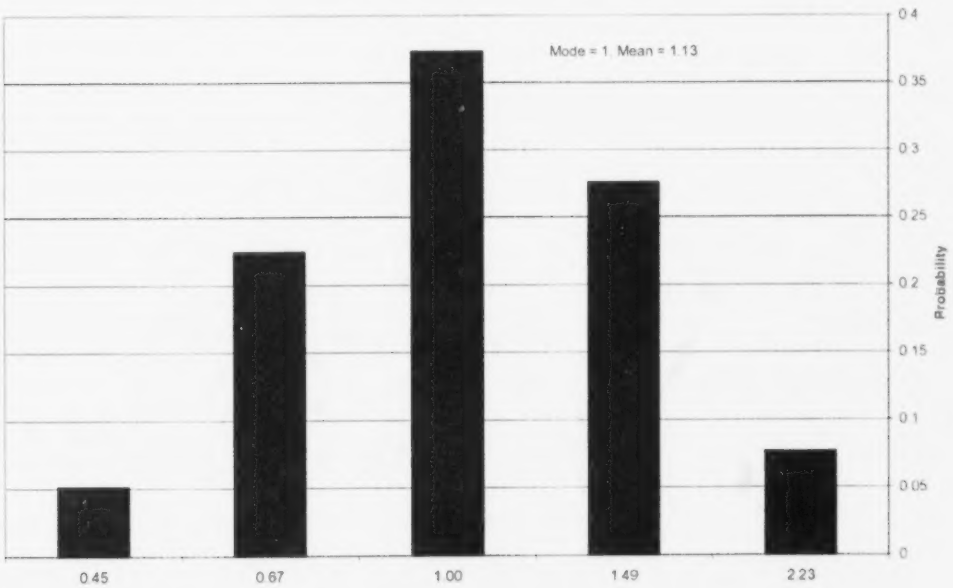


Figure 6. Log-normal probability distribution, mu = 0.03, sigma = 0.2



Box 1. Estimating Volatility from Historical Data

Volatility is defined as the annualized daily standard deviation of returns. For example, daily oil prices (R_t) in 2001 can be transformed into daily returns (X_t=lnR_t-lnR_{t-1}) as shown in the Figure below. A sample variance of daily returns can then be computed using the following traditional formula:

$Var = \sum_{t=1}^{T} \frac{(X_t - \overline{X})^2}{T-1}$. Volatility is then defined as the square root of the annualized variance of daily returns, that is:

$$\sigma = \sqrt{\left(\sum_{t=1}^{T} \frac{(X_t - \overline{X})^2}{T-1} \right)} * \text{Number of trading days in a year}.$$

The variance of daily returns in the oil industry in 2001 is 0.00086. The *p.a.* volatility is thus given by: $\sigma = \sqrt{0.00086 * 260} = 0.47$.

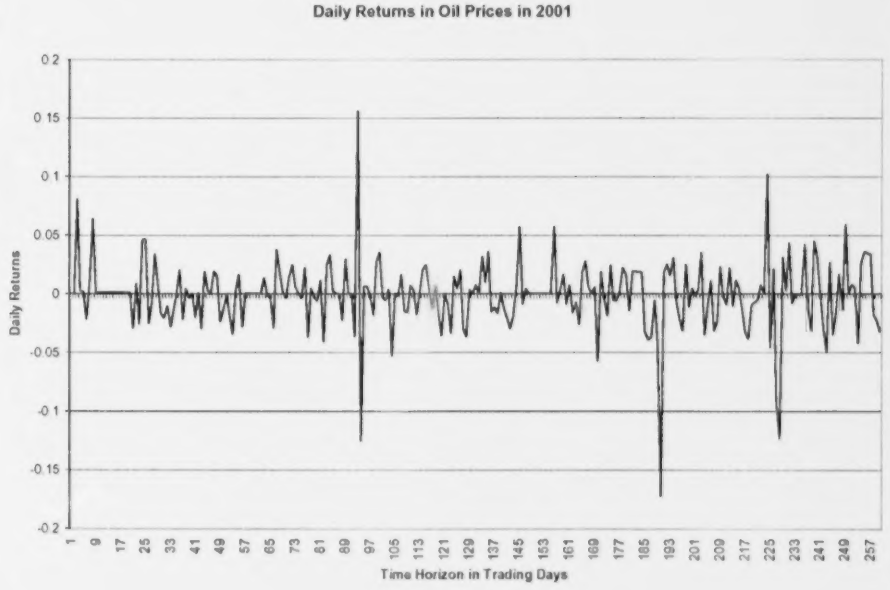


Table 2. Annual flow of operating revenues with positive trend

Time Horizon	Year 1	Year 2	Year 3	Year 4	Year 5	Present value[13]	Net present value
Annual flow of expected operating revenues	R	$R(1+\mu)$	$R(1+\mu)^2$	$R(1+\mu)^3$	$R(1+\mu)^4$	$R/(r-\mu)$	$R/(r-\mu)-K$
Marshallian's critical value, R=1; K=50;R =0.05; μ=0.03	1	1.03	1.06	1.09	1.13 (Mean in Figure 6)	1/(0.02)	0

With this background on the "parameters" of the model, it can be shown (see Appendix A2) that the distance between the Marshallian and the optimal critical values is given by:

$$E = M\left(\frac{\lambda_2}{\lambda_2 - 1}\right) \tag{3}$$

where:

$$\lambda_2 = -\frac{\mu}{\sigma^2} + \frac{1}{2}\left(1 + \sqrt{\frac{4\mu(\mu-\sigma^2)}{\sigma^4} + 1 + \frac{8r}{\sigma^2}}\right) > 1, \tag{4}$$

where σ is the volatility of the project (σ = 0.2 in Figure 1 for example), μ is the trend (μ = 0 in Figures 1 and 5, μ = 0.03 in Table 2), and r is the discount rate.

Now consider a discount rate of 5 % per year (r=0.05) and a volatility and a trend for the project set at σ = 0.2 and μ=0. We can then compute that $\lambda_2 = 2.15$ and thus $E = 1.86*M$. This means that if the investor considers that investment is at its critical value according to the traditional theory (say, M=1), then the net revenue would have to rise to nearly double the level (E = 1.86) before waiting ceases to be optimal. This example helps to illustrate Dixit's argument, which implies that, for "reasonable" parameters values, the

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[13] The present value (to Year 0) of the annual flow of operating revenues in Table 2 is given by:

$$PV = \frac{R}{1+r} + \frac{R(1+\mu)}{(1+r)^2} + \frac{R(1+\mu)^2}{(1+r)^3} + \ldots = \frac{R}{1+r}\underbrace{\left(1 + \frac{1+\mu}{1+r} + \left(\frac{1+\mu}{1+r}\right)^2 + \ldots\right)}_{=1/\left(1-\frac{1+\mu}{1+r}\right)\,(\text{if } (1+\mu)/(1+r)<1 \Rightarrow \text{if } r>\mu)} = \frac{R}{1+r}\frac{(1+r)}{(1+r-1-\mu)}$$

$$= \frac{R}{r-\mu}$$

potential revenues of a project would have to rise to nearly double the level that ensures a positive net worth before waiting ceases to be optimal.

We can also represent this argument in terms of the rate of return. The traditional theory asserts that the rate of return of a project at the Marshallian trigger ($R/K = M/K$) is equal to the opportunity cost of capital or discount rate, r. The theory of the value of waiting asserts, however, that the rate of return must be at least equal to $R/K = E/K$ before investing. Using equation (3), this implies an adjusted required rate of return or adjusted discount rate given by:

$$r' = \frac{E}{K} = \frac{M}{K}\frac{\lambda_2}{(\lambda_2 - 1)} = r\frac{\lambda_2}{(\lambda_2 - 1)} \tag{5}$$

In our example above, we find that $r' = 9.3\%$, which is significantly larger that the discount rate $r = 5\%$. This means that with the appropriate discount rate or opportunity cost of capital set at 5%, we would not see firms investing before that the rate of return of the project reaches 9.3%. This may explain why, according to casual observation, firms invest in projects which they expect to yield a return at least twice as large as the cost of capital.

From equation (2) we observe that in order to account for the full cost of immediate investment, we need to incorporate the value of waiting into the net present value formula. Alternatively, we can use the adjusted required rate of return such that, substituting (5) into (1), we obtain:

$$NPV_v = \frac{R}{r'} - K = \frac{R}{r\left(\frac{\lambda_2}{\lambda_2 - 1}\right)} - K. \tag{5bis}$$

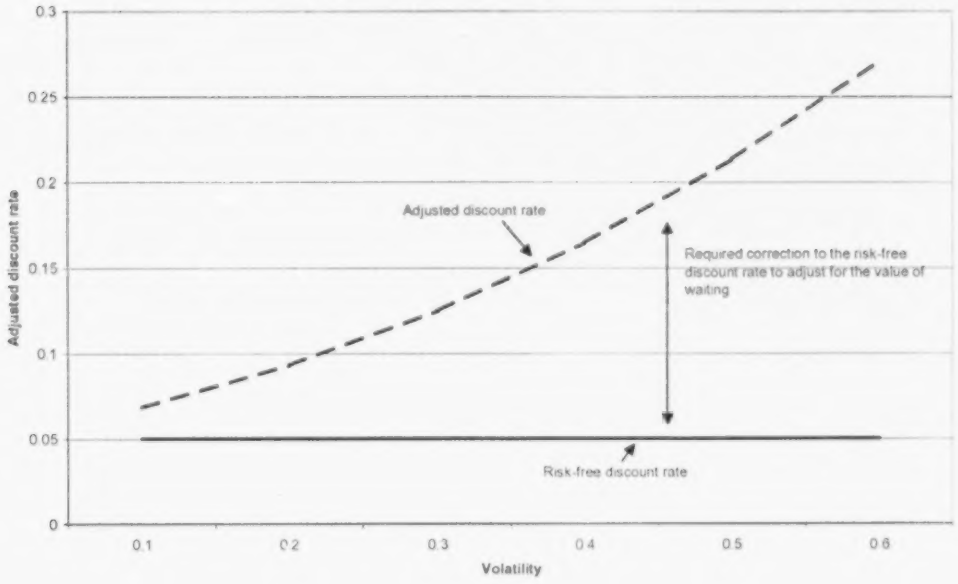
Although this adjustment in the net present value formula seems "similar" to the method of adding a risk premium to the risk-free discount rate, we should recall that the agent in this case is assumed to be risk neutral. In other words, the adjustment in the discount rate reflects value of waiting consideration and not risk aversion.

Increased volatility

What happens if uncertainty increases? It is intuitive that greater uncertainty means a higher value of waiting. If uncertainty is high, then setting a high critical value for E before taking action may avoid some very bad outcomes, if in fact the flow of net revenues of the project were very low. Pursuing our example above, if we increase the volatility parameter by 50% setting $\sigma = 0.3$, but keep all other parameters unchanged, this results in $\lambda_2 = 1.66$. In terms of the adjusted discount rate, we observe that r' rises to 12.6%. This means that with the appropriate discount rate or opportunity cost of capital set at 5%, we would not see firms investing in projects before that the rate of return

reaches 12.6%! Figure 7 shows the adjustment in the discount rate for various levels of volatility.

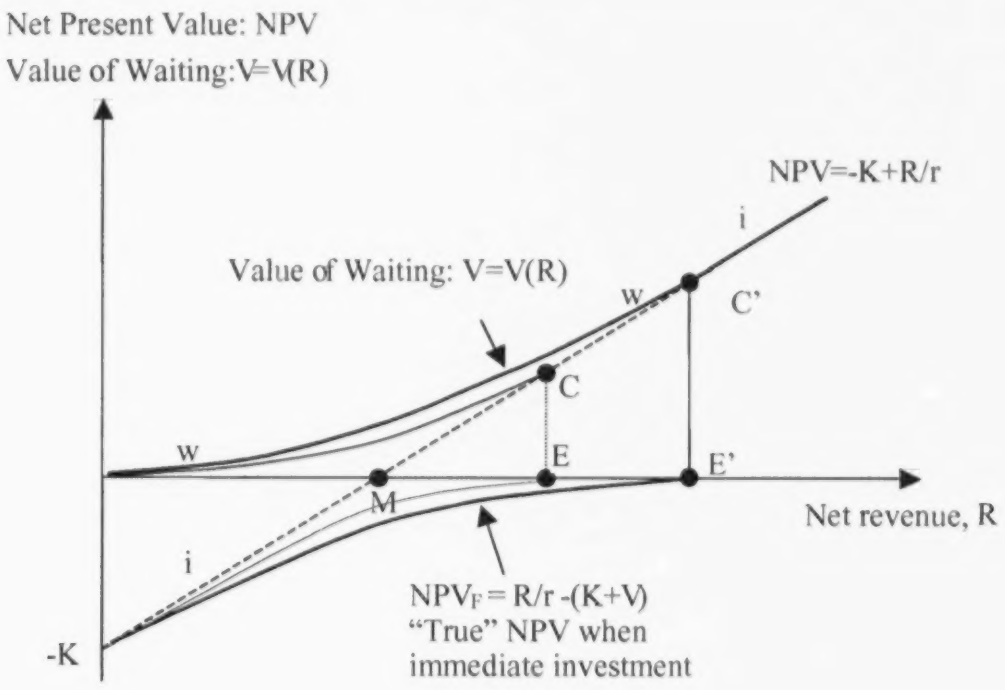
Figure 7. Adjusted discount rates for different levels of volatility



In terms of the optimal revenue trigger, its value rises to $E = 2.5*M$. Thus an increase in the volatility of revenues would increase inertia in investment decisions. Expected net revenues would now have to increase by 2.5 times (instead of 1.8 times) before waiting ceases to be optimal.

Figure 8 shows the impact of increased volatility on the schedules for the value of waiting and the adjusted net present value of immediate investment (*i.e.*, when considering the value of waiting). The optimal trigger moves from point E to point E'.

Figure 8: Increased volatility, value of waiting and adjusted net present value



Average time to reach the optimal critical value

We can proceed further and ask how long it would take before reaching the optimal trigger *E* conditional on whether we are currently at the Marshallian trigger. This can be solved analytically using the backward/forward Kolmogorov equations. Alternatively, we can simulate a large number of realisations of the process for the net revenue underlying Figure 1. Our average results are shown in Figure 9.

Figure 9. Probability of reaching the optimal trigger (conditional on being at the Marshallian trigger)

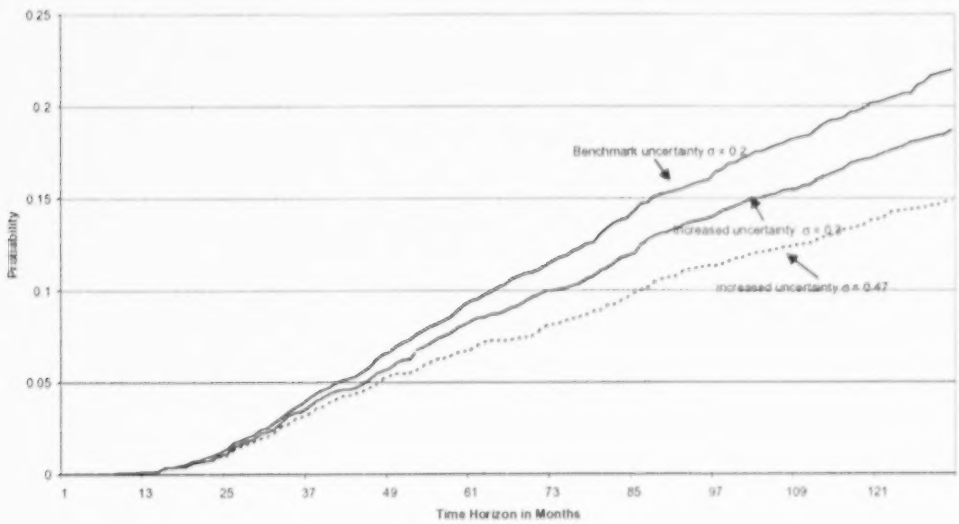


Figure 9 illustrates that uncertainty has a large impact on the inertia in investment decisions. The upper path shows that if a project is evaluated as being at its Marshallian value in the current time, there is only a small probability of 22% that the present value of the project reaches the optimal trigger 10 years later and thus that the investor implements this project.

When evaluating the impact of a shock, such as an increase in volatility, however, we must realise that investors were already facing an uncertain environment. Hence, the appropriate way to gauge the impact of such a shock is to ask how inertia is increased if the uncertainty is increased from its current level. Figure 9 shows that an increase in volatility by 50% (from 0.2 to 0.3) would lower the probability of reaching the optimal trigger ten year from now by only 3% points (from 22% to 19%). Another way to interpret Figure 9 is to say that on a 10-year time interval, an increase in volatility by 50% could delay investment by about 2 years, that is, we need to wait 10 years instead of 8 years to observe the same quantity of new investment in the economy.

Finally, the lower path shows that if volatility is more than doubled (from 0.2 to 0.47), we would need to wait 10 years instead of 6 years in order to observe the same quantity of additional investment.

4. Kyoto and Investment Behaviour

Do studies underestimate the negative impact of Kyoto on investment by neglecting the value of waiting described in Section 3? The answer to this question fundamentally

depends on whether Kyoto is viewed as a decrease in the level of revenues or an increase in volatility of revenues, or a mixture of both.

Kyoto: A decrease in the level of net revenue?

To demonstrate why this is the case, it is helpful to express the net revenue function of an investment project for a firm, which is a large final greenhouse gas emitter, with or without the Kyoto Protocol, as:

$$R_{without} = PX - \sum_i w_i L_i \tag{6}$$

$$R_{with} = PX - \sum_i w_i L_i - P_E Q_E \tag{7}$$

where X and P are the good's production and price of the firm, $\sum_i w_i L_i$ is total input cost excluding the emission permit cost, Q_E represents the number of tons of CO_2 in the production process of X, and P_E is the price of a permit to emit one ton of CO_2. Hence, implementing the Kyoto protocol entails an additional cost for the firm given by $P_E Q_E$ referred to as the emission cost, which reduces the net revenue of the firm.[14] It is worth noting, however, that companies will not necessarily be responsible for 100% of the emission cost. The government has considered free permit allocation up to 85% of Q_E.

In both the traditional Marshallian theory of investment and the value of waiting theory, a decrease in the annual flows of revenues would entail a lower net present value of investment opportunities.[15] In general, as shown in Section 3, the traditional theory overestimates the net present value of a project because, by neglecting the value of waiting V, it does not take into account the full cost of immediate action ($K+V$). A decrease in the annual flows of revenues will not modify the conclusion that the net present value of a project remains overestimated in the traditional theory by the option value of waiting V. (This can also be viewed by comparing the two net present values curves in Figure 4).

Moreover, another issue arises relating to the *change* in the net present value of investment opportunities due to a change (a fall) in expected revenues due to Kyoto. Suppose that a modeller is asked to compute the change in the net present value of investment opportunities due to lower anticipated revenues. If this person is unaware of the theory of the value of waiting, then they will use a net present value formula as given in equation (1) above, and compute the change in the net present value due to a given

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[14] Obviously, there are many general equilibrium impacts that we do not mention. For example, the price of the good, P, is likely to fall as demand falls reflecting a substitution to less polluting energy. This would further reduce revenues in the industry.

[15] For example, a recent briefing note by Lehman Brothers claims that the estimated present value of particular oil sands projects in Canada could in fact be reduced from 4%-6% (based on a 85% free permit allocation assumption) to 30%-40%, if companies are responsible for 100% of the emissions cost.

decrease in the revenue R. The modeller would observe that the net present value falls according to: $\frac{dNPV}{dR} = \frac{1}{r}$. If they also have an analytical function linking the net present value to the level of investment, the final impact on investment can be computed.

On the other hand, if the modeller is aware of the theory of the value of waiting, they would start from equation (2) and conclude that the net present value falls according to: $\frac{dNPV_v}{dR} = \frac{1}{r} - \frac{dV}{dR}$. It can be shown that $0 \leq \frac{dV}{dR} \leq \frac{1}{r}$ over the relevant range [0, E] (as is observed from Figure 4). Thus, for a same fall in R, the change in the net present value taking into account the value of waiting will be smaller than when the value of waiting is omitted. A smaller reduction in the net present value implies that investment would be affected to a lesser extent by a decrease in R than the traditional modeller would think.

In conclusion, following a fall in expected revenues due to Kyoto, a modeller who neglects the option value of waiting is likely to *overestimate* the fall in the net present value of potential investment opportunities and thus would *overestimate* the negative impact on investment.

We represent the overestimation of the fall in the net present value following a fall in revenues by ΔR as: [16]

$$(8) \qquad \left(\frac{dNPV}{dR} - \frac{dNPV_v}{dR}\right)\Delta R = \left(\frac{dV}{dR}\right)\Delta R = \lambda_2 \frac{V}{R}\Delta R$$

Figure 10 illustrates graphically this overestimation at the marginal investment in both "worlds", that is at point M in the traditional theory and at point E in the theory that includes the value of waiting.

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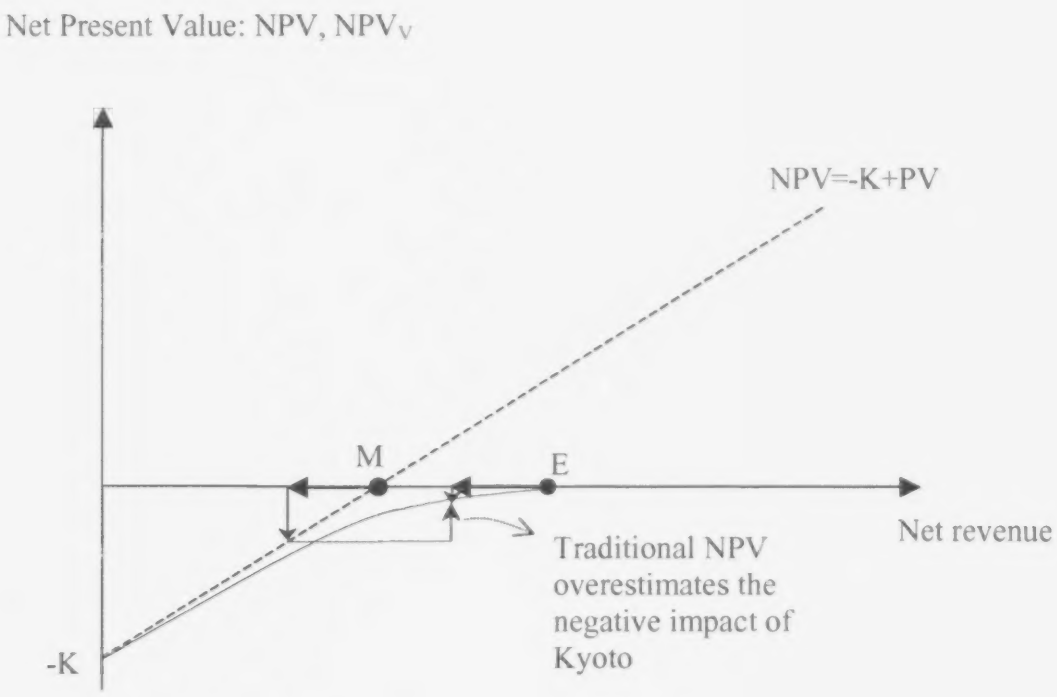
[16] In the Marshallian theory and for a project with no trend ($\mu = 0$), $\frac{dNPV}{dR} = \frac{1}{r}$, whereas in the option value theory: $\frac{dNPV_v}{dR} = \frac{1}{r} - \frac{dV}{dR}$, where $\frac{dV}{dR} \geq 0$. Graphically these expressions are the respective slopes of the two net present value curves in Figure 4. Hence the Marshallian theory overestimates changes in the NPV by $\frac{dV}{dR} \geq 0$. As shown in equation (20a) of the appendix, $\frac{dV}{dR} = \lambda_2 \frac{V}{R}$. This term tends to zero as R tends to zero. For subsequent higher values of R, it takes successively higher positive values to reach a maximum of $\frac{dV}{dR} = \frac{1}{r}$ when R equals the optimal trigger E. (For higher values of R, this term is not defined because the value of waiting falls to zero as soon as investment is realised.) Hence the overestimation of the change in the net present value in the Marshallian theory reaches a maximum of $1/r$ at the optimal trigger. For lower level of revenues the overestimation is reduced and gradually falls to zero when R falls to zero (*i.e.*, for extremely bad investment opportunities).

Figure 10: Kyoto, a decrease in net revenue?



Kyoto: An increase in volatility?

Large final emitters have often claimed that Kyoto would generate increased uncertainty in revenues, resulting from the requirement of buying green-house gas emission permits at variable price and unknown quantities.

This can be established using equations (6) and (7) above. Assuming a given level of production X, its associated tons of CO_2, Q_E, and a given input cost $\sum_i w_i L_i$, the variance of the net revenue in absence of Kyoto is essentially captured by the variance of the output price, that is:

(9) $$Var(R_{without}) = X^2 VarP$$

whereas considerations for Kyoto entails a variance given by:

(10) $$Var(R_{with}) = X^2 VarP + Q_E^2 VarP_E - 2XQ_E Cov(P, P_E)$$

Assuming initially that the covariance between the price of the good and the permit price is zero, Kyoto would increase the variance of an investor's net revenues by $Q_E^2 VarP_E$. The order of magnitude of $Q_E^2 VarP_E$ with respect to $X^2 VarP$ is unclear. However, in absence of a market for emission permits, the additional variance may be perceived as quite large. Typical estimates reported in the Canadian Climate Change Plan set the price of the permit in a range between $10 and $50 per ton of CO_2. This type of uncertainty could significantly increase the variance of the expected revenues of current long-term investment opportunities that eventually would be affected by the Kyoto protocol. Investment decisions in 2008 and beyond would also be affected by the variance of the permit price, but this impact would probably be considerably lower than it is now, when a market for permits is effectively implemented and the range -- and thus the variance of the permit price -- is reduced.

Any measures that serve to reduce the term $Q_E^2 VarP_E$ would reduce the uncertainty faced by the large final emitters. This could be accomplished with a price cap on P_E, or a free permit allocation (*e.g.*, requiring that x% of Q_E --the tons of CO_2 emissions due to the production--is freely allocated, while (1-x)% must be purchased on the market). Hence the policy response to the concerns of the large final emitters seems appropriate when analysed within the theory of investment under uncertainty.[17] Another possible measure could be the implementation of a forward/futures market for permits well before the official launch of Kyoto in 2008, which could help reveal the implied volatility of the permit prices and help mitigate the amount of uncertainty. Economists have previously flirted with the idea of a derivative (forward) market for a non-existent product (*i.e.*, the emission permit). For example, Obstfeld (1997) suggested a forward market for the euro in the period preceding the euro.[18]

We initially assumed that the covariance term between the price of the good and the permit price, $2XQ_E Cov(P, P_E)$, equalled zero. In reality, the covariance term could be either positive or negative depending on the nature of shocks. For example, shocks that lower the demand for the good would likely reduce the demand for emission permits, implying a decline in the price of the good and the price of permit. This should also mitigate the impact of the term $Q_E^2 VarP_E$ on the variance of the net revenues. On the other hand, shocks that lower the supply of the good would imply an increase in its price

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[17] Capping the permit price entails, however, potential fiscal costs because the government would have to pay the difference between market and capped prices.

[18] This paper analysed the constraints that the European Union law placed on the 1 January 1999 choices of irrevocably fixed conversion rates between the euro and the currencies of EMU members states. EU legislation, notably the Maastricht Treaty, required that the bilateral currency conversion factors implied by the 1 January 1999 choices equalled closing market exchange rates on 31 December 1998. Given that legal constraint, there were several strategies for choosing the relative prices of EMU member currencies against the euro. Unfortunately, most of these had potentially damaging side effects. One approach suggested by Obstfeld, based on contingent euro forward contracts with value dates at the start of Stage 3, allowed a highly credible pre-announcement of the bilateral currency conversion factors to be set at the start of EMU.

and a decrease in the price of permits, leading to a negative covariance and adding to the impact of the term $Q_E^2 VarP_E$ on the variance of the net revenues.

Suppose that a modeller is asked to compute the change in the net present value of investment opportunities following a higher volatility of revenues due to Kyoto. If they are unaware of the theory of the value of waiting, but do take into consideration that investors are risk neutral (which is the assumption done throughout this paper), the modeller will again use the net present value formula given in equation (1) above, and compute the change in the net present value due to a given increase in the volatility σ and will find that the net present value remains unchanged because it is independent of σ and thus: $\frac{dNPV}{d\sigma} = 0$. The modeller would thus conclude that an increase in volatility due to Kyoto would have no impact on the level of investment in Canada.

On the other hand, if the modeller is aware of the theory of the value of waiting then they will start from equation (2) and conclude that the net present value falls according to:

$$(11) \quad \frac{dNPV_v}{d\sigma} = -\frac{dV}{d\sigma} = -\frac{dV}{d\lambda_2}\frac{d\lambda_2}{d\sigma} = \underbrace{-V\left[\ln\left(\frac{R(\lambda_2 - 1)}{rK\lambda_2}\right)\right]}_{=\frac{dV}{d\lambda_2}<0}\underbrace{\left[\frac{-2r}{\sigma^3\left(\lambda_2 - \frac{1}{2}\right)}\right]}_{=\frac{d\lambda_2}{d\sigma}<0} < 0.$$ [19]

Thus, an increase in volatility lowers the net present value of investment opportunities, while it remains unchanged based on the traditional theory. Hence, the modeller who neglects the value of waiting would *underestimate* the fall in the net present value and therefore underestimate the potential negative impact on investment following a raise in volatility.

We represent the underestimation of the present value following an increase in volatility by $\Delta\sigma$ as:

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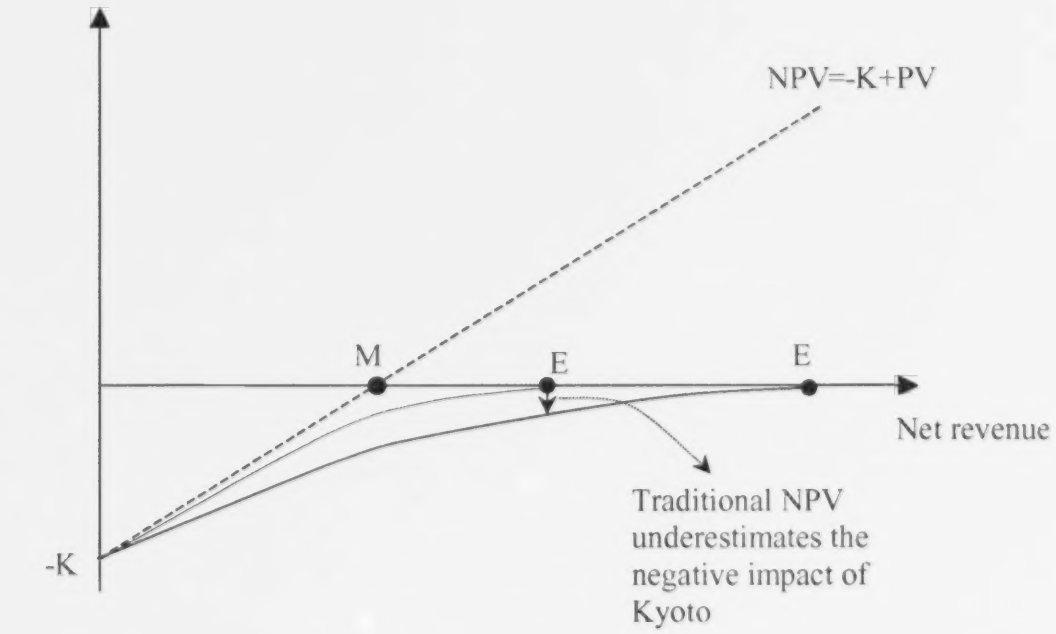
[19] Equations 14c and 20b of the Appendixes show that $\frac{d\lambda_2}{d\sigma} < 0$, and that $\frac{dV}{d\lambda_2} < 0$ if $\frac{R}{r} < \left(\frac{\lambda_2}{\lambda_2 - 1}\right)K$. Given that $\lambda_2 > 1$, this will always be true for the range of revenues [0,E). At point $E = rK\left(\frac{\lambda_2}{\lambda_2 - 1}\right)$, the derivative $\frac{dV}{d\lambda_2}$ equals zero. For revenues larger than E, the derivative is positive. Recall, however, than the value of waiting is not defined for revenues higher than E, and hence we can unambiguously state that $\frac{dV}{d\lambda_2} < 0$ and thus that $\frac{dV}{d\sigma} > 0$ for investment opportunities.

$$(12) \quad \left(\frac{dNPV}{d\sigma} - \frac{dNPV_v}{d\sigma}\right)\Delta\sigma = V\left[\ln\left(\frac{R(\lambda_2 - 1)}{rK\lambda_2}\right)\right]\left[\frac{-2r}{\sigma^3\left(\lambda_2 - \frac{1}{2}\right)}\right]\Delta\sigma > 0$$

Recall that all results have been derived under risk neutrality. If equations (1) and (2) were adapted to take into account risk aversion with the addition of a risk premium or using certainty equivalent arguments, the net present value would fall further in both "worlds" but the modeller who neglects the value of waiting would continue to underestimate the fall in the net present value by the same amount. It is worth repeating that the value of waiting is a concept unrelated to risk aversion but reflects a selective reduction of risk over time.

Figure 11 illustrates graphically the underestimation given in equation (12).

Figure 11: Kyoto, an increase in volatility?



Kyoto: An increase in volatility and a decrease in the net revenue?

On the other hand, we said earlier that a modeller who neglects the option value of waiting is likely to overestimate the fall in the net present value of investment opportunities following a fall in expected revenues due to Kyoto. If Kyoto is modelled as a mixture of lower expected revenues and increased volatility, it could be the case that the errors associated with neglecting the value of waiting would cancel out, but this would generally not be the case.

The modeller who neglects the value of waiting will globally underestimate the fall in the net present value of investment opportunities and thus underestimate the negative impact on investment due to Kyoto if, [using (8) and (12)]:

$$\underbrace{\overbrace{\left[\lambda_2 \frac{V}{R}\right]}^{>0}\Delta R}_{\text{overestimation of fall of NPV due to a decrease in R by } \Delta R} < \underbrace{\overbrace{V\left[\ln\left(\frac{R(\lambda_2-1)}{rK\lambda_2}\right)\right]}^{>0}\left[\frac{-2r}{\sigma^3\left(\lambda_2-\frac{1}{2}\right)}\right]\Delta\sigma}_{\text{underestimation of fall of NPV due to an increase in volatility by } \Delta\sigma} \quad (13)$$

Inequality (13) depends on the extent to which Kyoto is modelled as a volatility shock or a shock to revenues. If Kyoto is essentially a volatility shock, then $\Delta R = 0$, inequality (13) holds and the modeller who neglects the value of waiting will underestimate the fall of the net present value of investment opportunities when gauging the impact of Kyoto and thus underestimate the negative impact of this shock on investment. If Kyoto is essentially a revenue shock, then $\Delta\sigma = 0$, inequality (13) does not hold and the modeller would overestimate the fall in the net present value of investment opportunities, and thus overestimate the negative impact on investment.

Inequality (13) also depends on the current potential revenue of investment opportunities. If R is such that firms are indifferent between investing or not according to the Marshallian theory, then $NPV = 0$ such that $R = rK$. Substituting R by rK in (13) leads to the inequality given by:

$$\frac{\lambda_2}{rK}\Delta R < \left[\ln\left(\frac{(\lambda_2-1)}{\lambda_2}\right)\right]\left[\frac{-2r}{\sigma^3\left(\lambda_2-\frac{1}{2}\right)}\right]\Delta\sigma \quad (14)$$

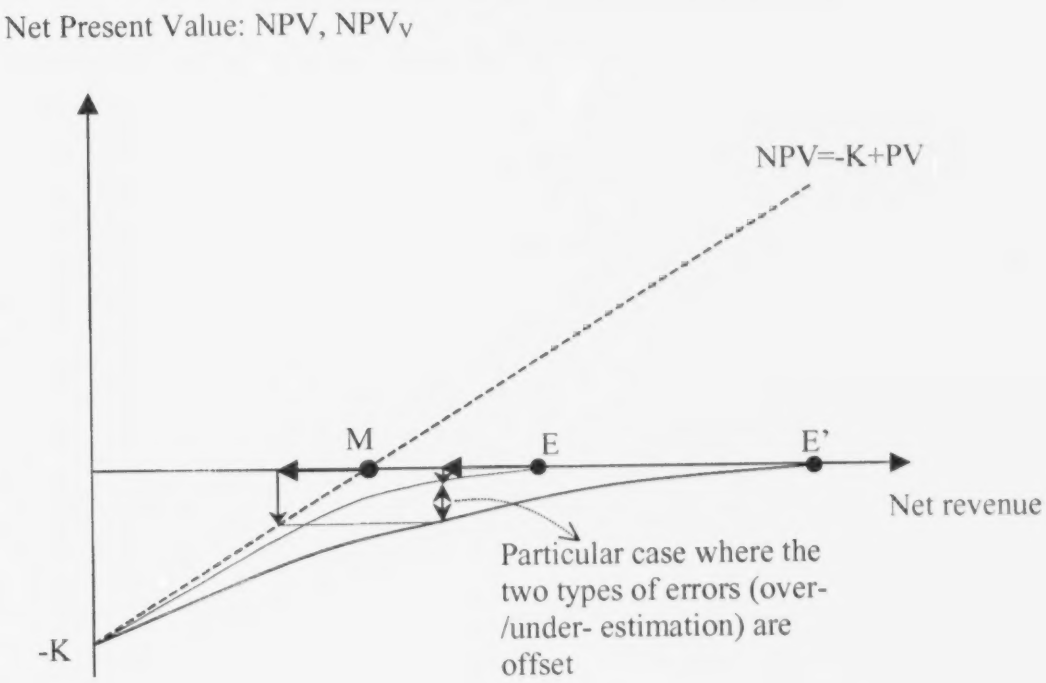
Substituting the parameter values given in Section 3, in particular: r=0.05, σ=0.2, λ_2 =2.15, K=20, leads to the following inequality:

$$\Delta\sigma \succ 0.454\Delta R \quad (15)$$

This means that the modeller would overestimate the net present value of investment opportunities as soon as the increase in the volatility parameter is larger than 0.454 times the decrease in revenues (in absolute sign). For example, if Kyoto is modelled as a 11% decrease in revenues in a specific sector (from $R = 1$ to $R = 0.9$), then an increase in the volatility parameter of 0.0454 (from $\sigma = 0.2$ to $\sigma = 0.2454$, *i.e.*, an increase in volatility of 23%) or more is sufficient to conclude that the modeller who neglects the value of waiting would underestimate the fall in the net present value of investment opportunities and thus underestimate the negative impact on investment.

Figure 12 illustrates a special case where both types of errors would offset.

Figure 12: Kyoto, an increase in volatility and a decrease in revenues?



Hints on How to Model the Value of Waiting in a General Equilibrium Model

It is difficult to proceed much further than the above analysis in terms of effective impact on investment levels of say, Kyoto, without recourse to a computable general equilibrium model (CGE). However, even the most recent generation of CGE models usually assumes away uncertainty such that these models are unable to take into account the concept of value of waiting.

A useful starting point toward modelling this concept would be to introduce the value of waiting into the investment function by modifying the net present value formula (as was shown in Section 3), or by modifying the Tobin'q concept usually built in CGE models to model the investment decision.

Tobin (1969) introduced a magnitude q, defined as the ratio of the "value of existing capital goods" to "their current reproduction cost". If this ratio exceeds unity, a firm could increase its market value by increasing its capital stock. Hence, we should see a firm investing when the value of q is above 1.

What is relevant for the investment decision is the effect of the next project on the value of the firm. When a project of present value $\frac{R}{r}$, of option value V, and sunk cost K, is considered, the value of the firm should increase by at least $K + V$ to be adopted. Tobin's q should therefore be defined in the context of investment under uncertainty as:

$$q = \frac{R/r}{K+V} \tag{16}$$

The appendix derives the analytical expression for V and shows that at the optimal trigger E, q takes the value: $q|_E = \frac{E/r}{K + E/r - K} = 1$, and hence at E, the marginal investment is justified according to the Tobin's q. On the other hand, at the Marshallian trigger, we show that q takes the value: $q|_M = \frac{M/r}{K+V} < 1$, and the investment should not be undertaken.

Although it is obvious that the value of waiting modifies the dynamics from one steady state to another, it is less clear how steady states themselves are affected by this concept. This is left for future research to CGE modellers.

5. Conclusion

During the debate preceding its ratification, a number of commentators and industry representatives argued that the uncertainty created by ratifying the Kyoto Protocol would negatively impact investment. This has led some to argue that ratification would have a greater negative impact on investment than that usually estimated by modellers, as the models typically used do not fully account for uncertainty.

This paper finds that: 1) There is a formal justification for the idea that the uncertainty created by ratification would tend to delay investment; 2) Policies such as capping the permit price can be effective at reducing this effect (albeit at a potential cost to the government); 3) Nonetheless, it is not the case that typical models (Computable General Equilibrium or macroeconomic models) necessarily underestimate the negative impact of Kyoto on investment.

This last point relates to the implementation of Kyoto, which may not only increase the *variance* of expected revenues but may also decrease the *level* of expected revenues.

On the one hand, the fact that these typical models do not take into account the increased volatility of cash flows or revenues will lead them to systematically underestimate the negative impact of Kyoto on investment. On the other hand, uncertainty-free modelling will tend to overestimate the impact that a fall in the level of expected revenues would have on investment. When future revenues are uncertain, firms tend to place less weight on a change in the average expected value of revenue. Thus it is unclear that ignoring uncertainty leads to underestimating the impact that implementing the Kyoto Protocol would have on investment, particularly when the price cap and the free allocation of permits are considered.

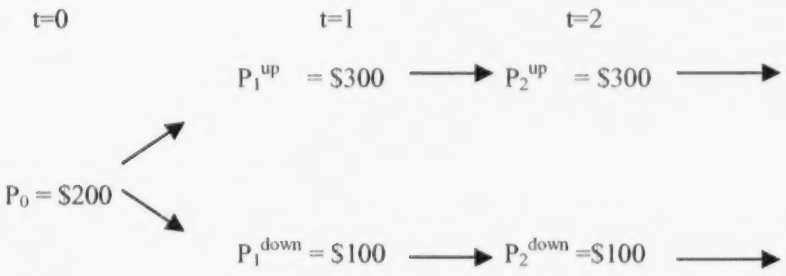
An Appendix describes the concept of value of waiting in an analytical framework, and shows how this concept changes our views on traditional concepts of net present value of an investment and the Tobin's q. It also illustrates the similarities between the analytical tools used in this approach also referred to as *real* option theory and those used in *financial* option theory.

Appendix A1: A Simple Example of the Value of Waiting[20]

Price uncertainty lasting two periods

Let us assume the following:

- A factory can be built instantaneously in year 0 at cost K_0=1600. (Note that if instead the factory is built in year 1, the cost remains K_1=1600.)
- The Factory will produce one widget per year, forever, with zero operating cost and sell at prices as follows:
- Price of widget is P_0 = $200 currently (year 0)
- Price next year is P_1^{up} = $300 with prob. q= ½, P_1^{down}=$100, with prob. (1-q)= ½.
- For future years, uncertainty is resolved: Prices remain what they are in period 1



Note: $P_1^e = P_2^e = P_0 = \frac{1}{2}300 + \frac{1}{2}100 = 200$ where the subscript "e" indicates an expected variable.

The question now posed is whether the firm should invest now or wait a year?

Invest now or never (Traditional Marshallian theory)

According to the net present value formula and given a discount rate set at r=10%, we conclude that we should invest now because the net present value of this project is positive:

---

[20] Appendix A1 is largely based on Chapter 2 of Dixit and Pindyck (1994).

$$NPV_0 = -1600 + 200 + \frac{200}{1+r} + \frac{200}{(1+r)^2} + \ldots$$

$$NPV_0 = -1600 + 200 + \frac{200}{0.1} = 600 \succ 0$$

Invest now or later

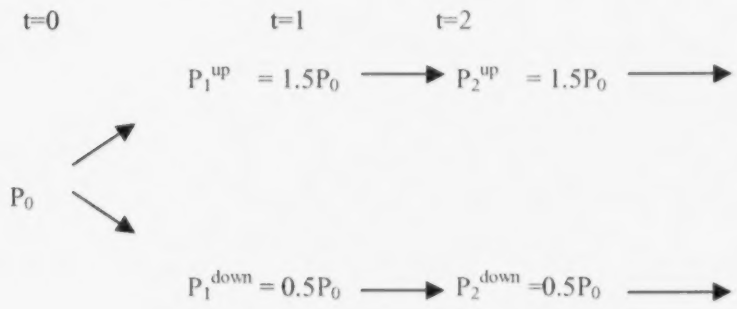
Suppose you wait and invest only if P goes up to 300, otherwise never invest.

$$V_0 = NPV_0 | waiting = \frac{1}{2}\left(-\frac{1600}{1+r} + \frac{300}{1+r} + \frac{300}{(1+r)^2} + \ldots\right) + \frac{1}{2}(-0+0)$$

$$V_0 = NPV_0 | waiting = \frac{1}{2}\left(-\frac{1600}{1.1} + \frac{300}{0.1}\right) = 773$$

Hence, if we wait a year before deciding (at time 1) whether to invest or not in the factory, the project's net present value today (at time 0) is $773, whereas it is only $600 if we invest in the factory at time 0. It is thus better to wait rather than to invest right away. The flexibility to delay is worth $773-$600=$173.

Generalisation to an unspecified initial price, P_0



In this case we have that:

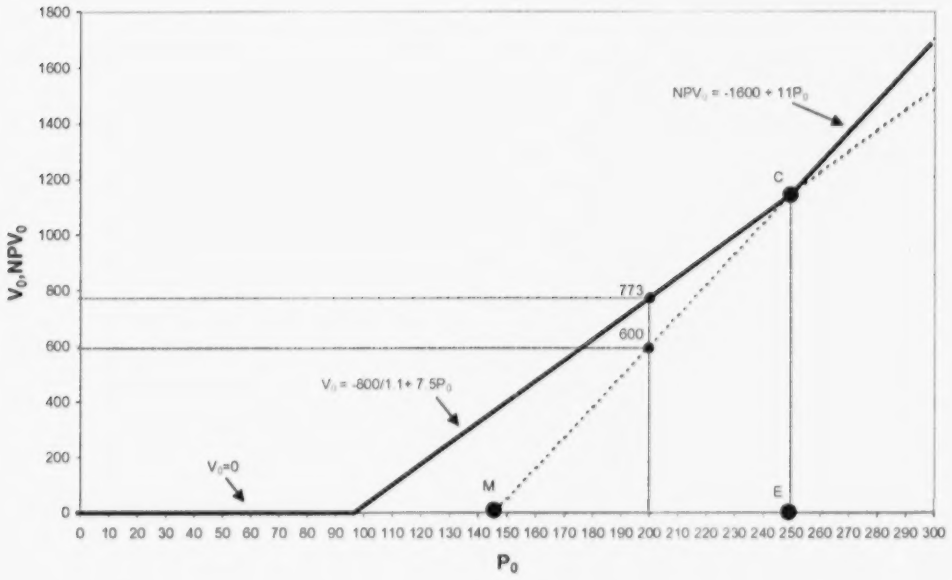
$$P_1^e = P_2^e = P_0 = \frac{1}{2}(1.5P_0) + \frac{1}{2}(0.5P_0)$$

$$NPV_0 = -K_0 + P_0 + \frac{P_0}{r} = -K_0 + P_0\frac{(1+r)}{r} = -1600 + 11P_0$$

$$V_0 = NPV_0 | waiting = \frac{1}{2}\left(-\frac{1600}{1.1} + \frac{1.5P_0}{0.1}\right) + \frac{1}{2}(-0+0) = -\frac{800}{1.1} + 7.5P_0$$

These relationships are plot in the figure A1.

Figure A1. Value of the Option to Invest as a Function of the Initial Price



The figure illustrates that for an initial price of $200, the value of waiting ($773) is larger than the value of immediate investment. Hence we should wait.

However, if P_0 *is low enough*, we might not want to invest even in the second period. In this case, the value of the investment opportunity, V_0 must be zero, that is:

$$V_0 = NPV_0 \mid waiting = \frac{1}{2}(-0+0)+\frac{1}{2}(-0+0) = 0$$

In particular, solving for P_0 such that $V_0 = 0 = -\frac{800}{1.1} + 7.5P_0$, we obtain that

$P_0 = \frac{800}{8.25} \approx 96.97$. Thus, if P_0 is less than 96.97, we know than an increase in price next period will not be sufficient to trigger investment. Thus the value of waiting is equal to zero.

Finally, *if P_0 is high enough*, we might want to invest right away and not wait any longer. This happens as soon as $NPV_0 > V_0$, that is, as soon as $-1600+11P_0 > -\frac{800}{1.1}+7.5P_0$, or, as soon as P_0>S249.

The important message is that the traditional criteria of net present NPV_0>0 is not valid in a world where the decision of investing is between now or later. In this world, the criterion is adjusted such that we should invest right away as soon as the present value of the investment opportunity (PV_0=$11P_0$) exceeds its *total* cost, $K_0 + V_0$, that is, as soon as $11P_0 \succ 1600+7.5P_0-\frac{800}{1.1}$, that is, as soon as P_0>S249.

The table below shows the optimal investment decision given the initial price.

Region	Option Value	Optimal Investment decision
$P_0 \leq 97$	V_0=0	Never invest
97<P_0<249	V_0= 7.5P_0-800/1.1	Invest in period 1 only if price has gone up
P_0>249	V_0=11P_0-1600	Invest in period 0

Hedging has no real impact on the net present value of the firm

Principle: To make sure that what may turn out to be different ($PV_1^{up} \neq PV_1^{down}$) is equalised by hedging strategy. For example:

$$PV_1^{up} = P_1^{up} + \frac{P_2^{up}}{1+r} + \frac{P_3^{up}}{(1+r)^2} = 300 + \frac{300}{1.1} + \frac{300}{(1.1)^2} + ... = 3300$$

$$PV_1^{down} = P_1^{down} + \frac{P_2^{down}}{1+r} + \frac{P_3^{down}}{(1+r)^2} = 100 + \frac{100}{1.1} + \frac{100}{(1.1)^2} + ... = 1100$$

To hedge, the investor should short-sell widgets at price S200 for each year in the future. This equalises the flows of revenues across state of the world in future periods. Indeed, when the price = S300, short-selling at S200 is a loss of S100 for each year. When price = S100, short-selling at S200 is a gain of S100 for each year.

$$PV_1^{up}|hedged = 300 - 100 + \frac{300-100}{1.1} + \frac{300-100}{(1.1)^2} + ... = 2200$$

$$PV_1^{down}|hedged = 100 + 100 + \frac{100+100}{1.1} + \frac{100+100}{(1.1)^2} + ... = 2200$$

Hence, with hedging strategy, investing today has a net present value given by:

$$NPV_0 = -K_0 + P_0 + \frac{PV_1|hedged}{1+r}$$

$$NPV_0 = -1600 + 200 + \frac{2200}{1.1} = 600$$

In conclusion, we have a variant of the Modigliani-Miller theorem: Hedging is a form of financial policy that has no real consequences for investment decision or the value of the firm. Adding a hedging strategy to our investment decision at time 0 does not change the net present value of the project, and does not change our conclusion that it is still better to wait than to invest at time 0.

A "Bad News Principle"

It is the magnitude of the possible "bad news" that drives the incentive to wait. We can prove this by showing that the optimal trigger E in Figure A1 or in Figure 4 does not depend in any way on the size of an upward move in prices. It depends only on the size of a downward move and the probability of a downward move.

Suppose that the initial price is P_0, but in period 1 the price becomes (and stays there forever):

$P_1 = (1+u)P_0$ with probability q

$P_1 = (1-d)P_0$ with probability $1-q$

The traditional net present value is given by:

$$NPV_0 = -K + P_0 + q\left[\frac{P_0(1+u)}{r}\right] + (1-q)\left[\frac{P_0(1-d)}{r}\right]$$

$$NPV_0 = -K + P_0 + \frac{P_0}{r}[uq + 1 - d + dq]$$

The value of waiting, on the other hand is:

$$V_0 = NPV_0|\text{if waiting} = q\left[\frac{-K}{1+r} + \frac{P_0(1+u)}{r}\right]$$

Equalising both expressions to obtain the critical price that makes the value of waiting and investing equal (Trigger E in Figures A1 or 4), obtains:

$$P_0\big|_E = K\left(\frac{r}{1+r}\right)\left(\frac{r+(1-q)}{r+(1-q)(1-d)}\right)$$

Hence, as said above, this critical value does not depend on u, the size of an upward move. It only depends on the size of a downward move, d, and the probability $(1-q)$ of a downward move. Also, the larger is d, the larger is the critical price. It is the magnitude of the possible "bad news" that drives the incentive to wait.

Appendix A2: A Mathematical Treatment

A2.1 Deriving the net present value of an investment

Assume that the net revenue R, of an investment follows a geometric Brownian motion with drift:

$$dR = R\mu dt + R\sigma dz \tag{1}$$

where z is a standard Wiener process, μ is the drift rate of the proportional change in R (*i.e.*, dR/R), and σ^2 is the variance rate of the proportional change in R.

The present value of the flows of expected future revenues, $PV(t)$, is thus given by:

$$PV(t) = \int_{\tau=t}^{\infty} E_t\left[R(\tau)|R(t)\right] e^{-r(\tau-t)} d\tau, \tag{2}$$

where $E_t\left[R(\tau)|R(t)\right]$ is the current (time-t) expectation of the investment revenue at a future date, $R(\tau)$, conditional on the current information, $R(t)$. Indeed, given that a geometric Brownian motion is a particular type of Markov stochastic process, the past history of the process and the way in which the present has emerged from the past are irrelevant. The current information is therefore adequately described as $R(t)$. Using the properties of a geometric Brownian motion, the time-t expectation of the return at time τ is given by:

$$E_t\left[R(\tau)|R(t)\right] = R(t)e^{\mu(\tau-t)}. \tag{3}$$

Substituting equation (3) into (2) gives:

$$PV(t) = R(t)e^{(r-\mu)t}\int_{\tau=t}^{\infty} e^{(\mu-r)\tau} d\tau. \tag{4}$$

Assuming r > μ to ensure convergence of the expected present value of the revenues and integrating, obtains:

$$PV(t) = \frac{R(t)}{r-\mu}. \tag{5}$$

Assuming a sunk cost, K, that a firm has to pay immediately to invest, the **net** expected present value of the investment , $NPV(t)$, is therefore:

$$NPV(t) = \frac{R(t)}{r-\mu} - K. \tag{6}$$

If we set the drift rate, μ, equal to zero in equation (6), we get the net present value of the investment given in Section 3, equation (1), of the paper.

A2.2 Deriving the value of waiting

Consider the investment opportunity as an asset that is held for a small interval of time dt and that has a value which is a function of R, that is, $V=V(R)$, where R follows the geometric Brownian motion given in equation (1).

Using Itô's lemma gives:

$$dV = V'(R)dR + \frac{1}{2}V''(R)(dR)^2. \tag{7}$$

Substituting equation (1) into (7), and using stochastic calculus rules, [in particular, $(dR)^2 = R^2\sigma^2 dt$], obtains:

$$dV = V'(R)\left[R\mu dt + R\sigma dz\right] + \frac{1}{2}V''(R)R^2\sigma^2 dt. \tag{8}$$

The instantaneous expected rate of change in V (capital gains) is therefore given by:

$$\frac{E(dV)}{dt} = V'(R)\left[R\mu\right] + \frac{1}{2}V''(R)R^2\sigma^2. \tag{9}$$

In equilibrium, this should be equal to the instantaneous normal rate of return, rV, that we would require for holding this asset using the discount rate, r. We therefore get the following differential equation:

$$\frac{1}{2}R^2\sigma^2 V''(R) + R\mu V'(R) - rV(R) = 0. \tag{10}$$

which can be rewritten as:

$$V''(R)+\frac{2\mu}{\sigma^2 R}V'(R)-\frac{2r}{\sigma^2 R^2}V(R)=0. \tag{11}$$

General solution of equation (11)

Try a solution of the form $V(R)=R^{\lambda}$, such that: $V'(R)=\lambda R^{\lambda-1}$ and $V''(R)=(\lambda-1)\lambda R^{\lambda-2}$. Substituting in (11) obtains:

$$R^{\lambda-2}\left((\lambda-1)\lambda+\frac{2\lambda\mu}{\sigma^2}-\frac{2r}{\sigma^2}\right)=0, \tag{12}$$

that is, the characteristic quadratic equation that λ must satisfy is:

$$\left((\lambda-1)\lambda+\frac{2\lambda\mu}{\sigma^2}-\frac{2r}{\sigma^2}\right)=0,$$

or:

$$\lambda^2+\lambda\left(\frac{2\mu-\sigma^2}{\sigma^2}\right)-\frac{2r}{\sigma^2}=0 \tag{13}$$

The quadratic function $Q=\lambda^2+\lambda\left(\frac{2\mu-\sigma^2}{\sigma^2}\right)-\frac{2r}{\sigma^2}$ is an upward-pointing parabola (because the coefficient of λ^2 is positive) that goes to ∞ as λ goes to $\pm\infty$. The left-hand side of equation (13) is negative at $\lambda = 0$, when $r > 0$. It is also negative when $\lambda = 1$, provided that $r > \mu$, which we have assumed to ensure convergence of the expected present value of revenues. Hence, one root of equation (13) is negative, and the other one exceeds one. The two characteristic roots are given by:

$$\lambda_1=-\frac{\mu}{\sigma^2}+\frac{1}{2}\left(1-\sqrt{\frac{4\mu(\mu-\sigma^2)}{\sigma^4}+1+\frac{8r}{\sigma^2}}\right)<0 \tag{14a}$$

and:

$$\lambda_2=-\frac{\mu}{\sigma^2}+\frac{1}{2}\left(1+\sqrt{\frac{4\mu(\mu-\sigma^2)}{\sigma^4}+1+\frac{8r}{\sigma^2}}\right)>1 \tag{14b}$$

As we will need it in the text, we focus on the root λ_2 and ask how does it change if the parameter σ changes? We can show that for $\mu = 0$,

$$\frac{d\lambda_2}{d\sigma} = \frac{d\left(\frac{1}{2} + \frac{1}{2}\left(\sqrt{1 + \frac{8r}{\sigma^2}}\right)\right)}{d\sigma} = -\frac{2r}{\sigma^3\left(\lambda_2 - \frac{1}{2}\right)} < 0 \tag{14c}$$

The general solution of equation (11) is therefore:

$$V(R) = A_1 R^{\lambda_1} + A_2 R^{\lambda_2}, \tag{15}$$

where A_1 and A_2 are constants of integration to be determined with the help of two boundary conditions.

Boundary conditions

Consider at this point an exogenous trigger at $R(t) = E > 0$, where the firm is indifferent between investing now or delaying. As long as $R(t) < E$, the firm has no incentive to invest. When $R(t) > E$, the firm invests. (Later, we will show how to choose E such that it is also the optimal trigger.)

The first boundary condition lies on the notion that the value of waiting should go to zero as $R(t)$ is very low (that is, as $R(t)$ approaches zero). Indeed, in this case, the event of $R(t)$ climbing to E is unlikely except in the far future, and the discounted present value of that is quite small. To force that pattern on $V(R)$, and because λ_1 is the negative root, we must set $A_1 = 0$.

The second condition relies on the traditional value-matching and smooth-pasting conditions. In the first step, consider that $R(t)$ approaches E, such that the probability of reaching E in the very near future approaches one. The firm approaches the situation where it is indifferent between investing and waiting. This means that at E, the value of waiting, $V(E)$, must approach the net (present) value of investing. As a consequence, at $R(t) = E$, we have that [using equations (15) and (6)]:

$$V(E) = A_2 E^{\lambda_2} = \frac{E}{r - \mu} - K. \tag{16}$$

In the second step, we have to recognise that in order to maximise the value of waiting, the trigger must be pushed up to the point where the slope of w-w (in Figure 4) is equal to the slope of i-i. This is referred to as the smooth-pasting condition. This condition gives the optimal trigger, E. Thus, equalising the first derivative (with respect to R and evaluated at $R=E$) of $V(R)$ with the first derivative of NPV given in equation (6), we get another equation that must be fulfilled at the optimal trigger:

$$\lambda_2 A_2 E^{\lambda_2 - 1} = \frac{1}{r - \mu}. \tag{17}$$

We therefore have a system of two equations, (16) and (17), and two unknowns, the optimal trigger, E, and the constant of integration, A_2. Solving this system, we get that:

(18) $$A_2 = \frac{1}{(r-\mu)\lambda_2\left[(r-\mu)K\left(\frac{\lambda_2}{\lambda_2-1}\right)\right]^{\lambda_2-1}}$$

and:

(19) $$E = (r-\mu)K\left(\frac{\lambda_2}{\lambda_2-1}\right).$$

Substituting equation (18) into (15), and setting $A_1 = 0$ as discussed above, obtains:

(20) $$V(R) = \left(\frac{R}{(r-\mu)\lambda_2}\right)^{\lambda_2}\left(\frac{\lambda_2-1}{K}\right)^{\lambda_2-1}.$$

Equation (20) is the analytical expression for the curve w-w in Figure 4.

Using equation (20) (and setting μ=0), it can be shown that:

(20a) $$\frac{dV}{dR} = \left(\frac{\lambda_2-1}{K}\right)^{\lambda_2-1}\left(\frac{R}{r\lambda_2}\right)^{\lambda_2-1}\frac{1}{r} = \lambda_2\frac{V}{R} \geq 0.$$

This term tends to zero as R tends to zero. For subsequent higher values of R, it takes successively higher positive values to reach a maximum of $\frac{dV}{dR} = \frac{1}{r}$ as R equals the optimal trigger E. (For higher values of R, this term is not defined because the value of waiting falls to zero as soon as investment is realised.)

In the text we need to compute and sign the derivative of V with respect to σ. Using the chain rule in equation (20), (and setting μ=0), obtains:

$$\frac{dV}{d\sigma} = \frac{dV}{d\lambda_2}\frac{d\lambda_2}{d\sigma}$$

Given that $\frac{d\lambda_2}{d\sigma} < 0$ (derived in 14c), all we need is to derive and sign $\frac{dV}{d\lambda_2}$. We can easily established that:

$$\frac{dV}{d\lambda_2} = \frac{de^{\lambda_2 \ln\left(\frac{R}{r\lambda_2}\right)}}{d\lambda_2} e^{(\lambda_2-1)\ln\left(\frac{\lambda_2-1}{K}\right)} + \frac{de^{(\lambda_2-1)\ln\left(\frac{\lambda_2-1}{K}\right)}}{d\lambda_2} e^{\lambda_2 \ln\left(\frac{R}{r\lambda_2}\right)}$$

$$= \left[\ln\left(\frac{R}{r\lambda_2}\right) - 1\right] e^{\lambda_2 \ln\left(\frac{R}{r\lambda_2}\right)} e^{(\lambda_2-1)\ln\left(\frac{\lambda_2-1}{K}\right)} + \left[\ln\left(\frac{\lambda_2-1}{K}\right) + 1\right] e^{(\lambda_2-1)\ln\left(\frac{\lambda_2-1}{K}\right)} e^{\lambda_2 \ln\left(\frac{R}{r\lambda_2}\right)} \quad \text{(20b)}$$

$$= \left[\ln\left(\frac{R}{r\lambda_2}\right) + \ln\left(\frac{\lambda_2-1}{K}\right)\right] V$$

$$= \left[\ln\left(\frac{R(\lambda_2-1)}{r\lambda_2 K}\right)\right] V$$

Thus, given that V is always positive, $\frac{dV}{d\lambda_2} < 0$ if $\frac{R}{r} < K\left(\frac{\lambda_2}{\lambda_2 - 1}\right)$. Given that $\lambda_2 > 1$ (see 14b), this is always true for the relevant range of revenues [0, E). At the optimal trigger, E, (given by equation 19 and setting $\mu=0$) it can be shown that $\frac{dV}{d\lambda_2} = 0$.

A2.3 Relationship between the Marshallian trigger and the optimal trigger

The Marshallian trigger is the point where the net present value of future revenues of the investment is equal to zero, that is it is the value of *R(t)* such that:

$$\frac{R(t)}{r-\mu} - K = 0.$$

The Marshallian trigger is therefore given by:

$$M = (r-\mu)K. \quad (21)$$

The optimal trigger is given in equation (19). The simple relation between both triggers follows:

$$E = M\left(\frac{\lambda_2}{\lambda_2 - 1}\right). \quad (22)$$

As λ_2 is larger than 1, it follows that $\left(\frac{\lambda_2}{\lambda_2 - 1}\right) > 1$ and thus E > M. Thus the simple NPV rule is incorrect; uncertainty and irreversibility drive a wedge between the critical values E and M. The size of the wedge is the factor $\left(\frac{\lambda_2}{\lambda_2 - 1}\right)$. Even if the net present value of investing becomes slightly positive, the firm should postpone its entry as long as the

current revenues of the investment are not sufficiently "in the money", to use option pricing terminology. This will be the case at E, where the firm should invest (exercise its option).

Observe from equation (22) that as λ_2 increases and tends to infinity, the wedge factor tends to 1, E tends to M, so that the value of waiting tends to zero and the Marshallian analysis applies. On the other hand, as λ_2 decreases and tends to 1, the wedge factor tends to infinity, E will be further away from M, and the Marshallian analysis becomes misleading.

By observing equation (14b), we know that λ_2 decreases, therefore the wedge increases and the value of waiting for more information rises when the uncertainty (volatility) of future revenues is higher (higher uncertainty σ), and when the discount rate r is lower. When the trend in future revenues is higher (μ is higher), the value of waiting will generally (but not always) increase. See Figures A2 and A3.

Figure A2: Value of waiting: An increase in volatility

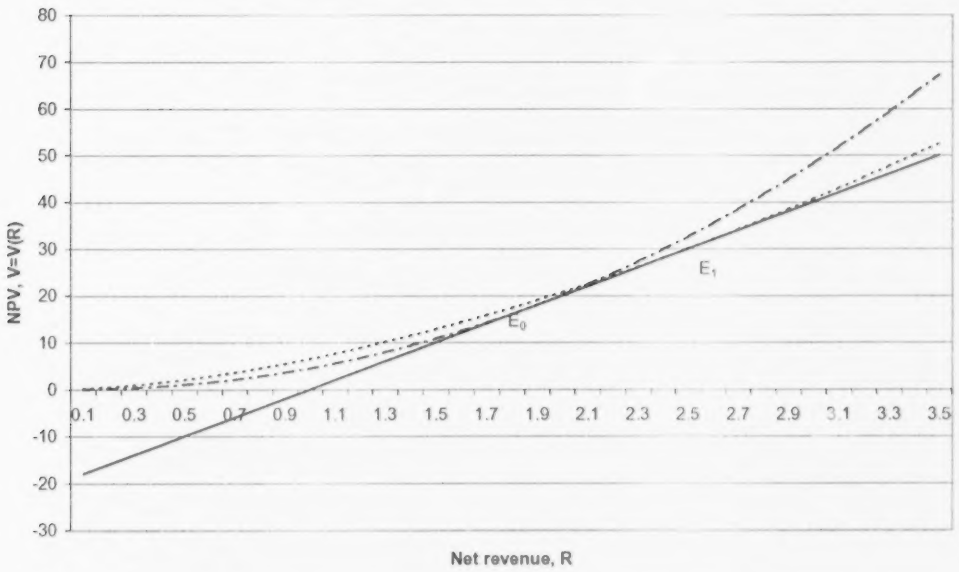
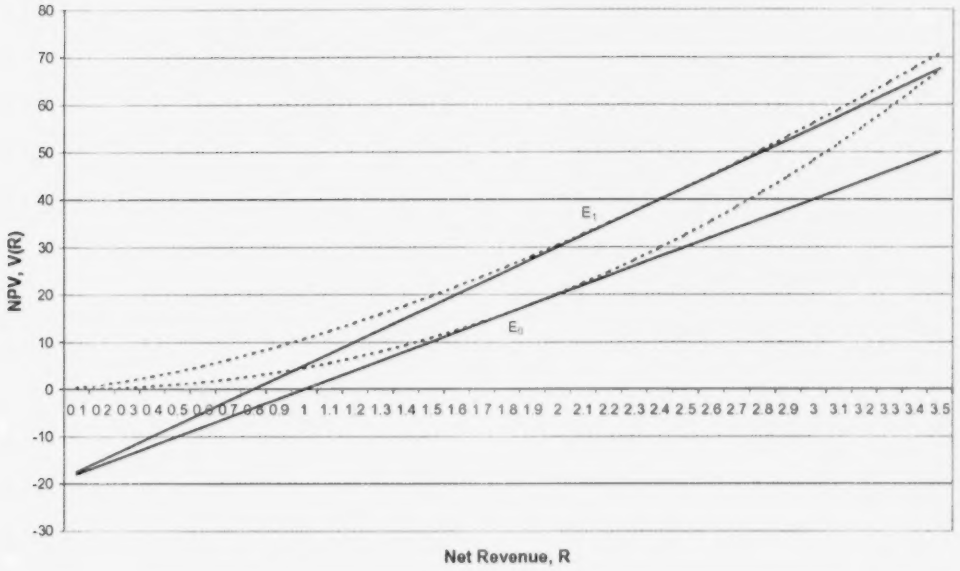


Figure A3: An increase in trend



A2.4 Relationship to Tobin's q

We saw that immediate action for the firm has an opportunity cost, the loss of the option to wait, *V(R)*. We must add this to the actual sunk cost, *K*, to get the full cost of immediate investment. Then, such action is justified as soon as the net present value of future revenues of the investment becomes equal to the full cost of immediate investment, that is when:

(23a) $$\frac{R}{r-\mu} = V(R) + K .$$

or:

(23b) $$\frac{R}{r-\mu} - K = V(R) .$$

As is demonstrated in Figure 4, this will occur when *R(t)* reaches *E*.

Tobin (1969) introduced a magnitude *q*, defined as the ratio of the "value of existing capital goods" to "their current reproduction cost". If this ratio exceeds unity, a firm can increase its market value by increasing its capital stock. Hence, we should see a firm investing when the value of *q* is above 1.

What is relevant for the investment decision is the effect of the next project on the value of the firm. When a project of present value $\frac{R}{r-\mu}$, of option value $V(R)$, and sunk cost K, is considered, the value of the firm should increase by at least $K + V(R)$ to be adopted. Tobin's q should therefore be defined in the context of investment under uncertainty as:

$$q = \frac{R/(r-\mu)}{K+V(R)} \tag{24}$$

Now, observe that at the trigger E, and given equation (16), q takes the value: $q|_E = \frac{E/(r-\mu)}{K+E/(r-\mu)-K} = 1$, and hence at E, the marginal investment is justified according to the Tobin's q. On the other hand, at the Marshallian trigger, we know that: $V(M) = A_2 M^{\lambda_2} \succ \frac{M}{r-\mu} - K$, as is obvious from Figure 4. Hence, q takes the value:

$q|_M = \frac{M/(r-\mu)}{K+V(M)} \prec 1$, and the investment should not be undertaken.

A2.5 Relationship to financial options

The opportunity to make a real investment is similar to an American call option – a right but not an obligation to buy a stock at a preset price (the exercise price). For the real investment considered above, the exercise price is the sunk cost K, of the project. If the option to invest is exercised, the firm acquires ownership of a stock that pays a dividend stream of expected value $\frac{R}{r-\mu}$. The net worth $\frac{R}{r-\mu} - K$, is called the "intrinsic value" of the option. It would not be optimal for the firm, however, to exercise this option when R remains below E because the value of the option to invest is higher than the intrinsic value of the investment. In other words, exercising the option at the moment its intrinsic value becomes positive is not optimal, because the option has a value of waiting referred to time value in financial option. One should wait until the time value falls to zero. The smooth pasting condition that helps to determine the optimal point of exercise (equation 17 above) has been developed in the literature of financial options.

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